ASTR 425 - Cosmology

Problem Set 2

Due: Jan 24, 2007

1. Robertson-Walker metric

   a) Show that the two forms of the Robertson-Walker metric (eq. 3.24 & 3.25) are equivalent for $\kappa = -1, 0, 1$. What is the physical meaning of x?

   b) What is the maximum volume of the Universe for $a(t) = 1$ and $\kappa = 1$ ($r \leq \pi R_0$)? Why should we care about the volume of a closed Universe?

   c) Compute the volume within a distance r for the $\kappa = -1$ case, and show that $V \simeq \frac{4\pi}{3} r^3$ for $r \ll R_0$.

   d) What technique(s) could be used to distinguish whether we live in a $\kappa = -1, 0, 1$ Universe?

2. Constant Hubble parameter

   Suppose that $H(t) = H_0$ (the expansion rate is constant for all time).

   a) What is $a(t)$? (Note: you don't need the Friedmann equation or Robertson-Walker metric, only the definition of H). Explain what this implies physically.

   b) What is the comoving distance to an object at redshift z?

   c) What is the proper distance to the same object at time t_{em}?

   d) Why might this solution be important for our Universe?

3. Show that the third Friedmann equation

$$a^3 \dot{p} = \frac{d}{dt}\left[a^3(c^2\rho + p)\right]$$

   or

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p/c^2)$$

is redundant by deriving it from the first two equations:

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4}{3}\pi G(\rho + 3p/c^2) \\ \left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho &= -\frac{\kappa c^2}{R_0^2 a^2} \end{aligned}$$