Name ____ Date ______________
Partners ____

Measuring the Mass of the Earth

Objective

To derive the mass of the Earth using direct measurement of the acceleration of an object at the Earth's surface.

Materials

- String
- Meter stick
- Stopwatch
- Marble (or other small, spherical object)

Introduction

Sir Isaac Newton changed the way in which humankind viewed the world. His laws describing the fundamental properties of physical reality took scientists from empirical work to mathematical logic. In particular, his description of gravity gave us a means to understand how we are bound to the Earth, how the Moon is bound to the Earth, how the Earth is bound to the Sun, and so on. We now understand how one planet can perturb another or how a distant cluster of galaxies can "pull" us across immense distances.

Using Newton's formulae and knowledge of the radius of the Earth and the universal constant of gravitation, we can determine the mass of the Earth. We will be using the following equations:

$F_{gravity} = G\frac{Mm}{R^2}$ (1)	$F = ma$ (2)	$x = \frac{1}{2}at^2$ (3)

where ***F*** is force, ***G*** (also known as **big G**) is the universal constant of gravitation:

$$G = 6.67 \times 10^{-11}\, m^3 / kg\, s^2 ,$$

M is the mass of the Earth, ***m*** is the mass of an object on the surface of the Earth, ***a*** is the acceleration of that object, ***x*** is the distance the object falls, and ***t*** is the amount of time it takes that object to fall that distance. Everything must be measured in mks units (meters, kilograms, seconds).

We can manipulate equations 1 and 2 (if we assume the acceleration in Eqn. 2 is due to the force of gravity, and thus $F = F_{gravity}$) to get the acceleration due to gravity in terms of $\boldsymbol{G}$, $\boldsymbol{M}$, and $\boldsymbol{R}^2$, or the mass of the Earth in terms of the acceleration, radius, and constant of gravitation:

$$a = G\frac{M}{R^2}, \quad M = \frac{aR^2}{G} \quad . \qquad 4)$$

Since we "know" ***R*** to be 6,378 km (=6,378,000 m; let's say we measured it somehow by observing the length and angles of shadows at different places on the Earth's surface) and **G** was fortunately measured for us by a physicist, all we need is ***a***, the acceleration of an object at the Earth's surface.

We don't need to know the mass of the object. (Why is this true?) Although any object can be dropped, use an object that will experience a minimum of air resistance. Drop the object from a height that is high enough to minimize the relative fraction of time it takes us to start and stop a stop watch, but not so high that air resistance starts to affect the results.

Procedure

Find a location suitable for dropping stones or marbles. Examples of locations include open stairwells, balconies, and rooftops. Measure the distance the objects will be falling, and make an estimate (**your best guess**) as to the accuracy of that measurement. Are you accurate to within a millimeter (1/1000 of a meter)? A centimeter (1/100 of a meter)? A few centimeters? A whole meter? Use a stopwatch with a precision of 1/100 of a second, and time how long each object takes to fall this distance.

Exercise

Record the times in the following chart:

Trial No.	Time (Sec)	Trial No.	Time (Sec)	Trial No.	Time (Sec)	Trial No.	Time (Sec)	Trial No.	Time (Sec)
1		5		9		13		17	
2		6		10		14		18	
3		7		11		15		19	
4		8		12		16		20	

1. Distance the object(s) fell (x): ________ meters; Uncertainty: ________ meters

Share this data with the other members of your team or class. For mathematical understanding, each team member should do the following calculations individually. Again, please show all calculations here.

2. Calculate the average time taken to fall the distance, x: Average time: _______ seconds

3. Calculate the uncertainty in this average (via an *approximate* method):

! Toss out the longest and shortest times
! Subtract the now-shortest time from the now-longest time and divide by 2

Uncertainty: _________ seconds

4. Square the value of the average time of fall: t^2 = _______ sec^2

5. We can manipulate Eqn. 3 above, and solve for the acceleration of the object:

$$a = 2x/t^2 = \_\_\_\_\_\_\_ \ m/s^2$$

6. Since the negative sign for the acceleration refers to the direction (down), we can disregard it when determining the mass of the Earth. Solve for the mass of the Earth:

$$M = \frac{aR^2}{G}, \text{ Mass} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ kg.}$$

Questions

1. Compare your value for the mass of the Earth to the true value of 6×10^{24} kg. That is, calculate the percentage difference:

2. How does your value for g compare to the actual value of 9.8 m/s^2 ? With this answer in mind, how does your measurement for ***g*** affect your derived value for the mass of the Earth?

3. Given your estimates for the uncertainties in the length and period, what is the range of values allowed for ***g*** at the surface of the Earth? That is, what are the lowest and highest values you find given the uncertainties of your experiment?

- Highest value comes from combining the longest possible length with the shortest possible period, given your uncertainties. Calculate the highest value for ***g*** :

- Lowest possible value comes from combining the shortest possible length with the longest possible period, given your uncertainties. Calculate the lowest value for ***g*** :

4. Does the real value for the mass of the Earth lie within your uncertainties? (You can figure this out without doing any additional calculations.) Explain.