ASTR 321-Fall 2009

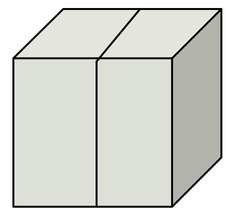
Problem set #3

Tides, Yarkovsky, Poynting-Robertson, Hill Spheres & Extra-solar planets

due Thursday Oct 22

1) If a comet passes too close to Jupiter it can break-up because the tidal force from Jupiter exceeds the comet's self-gravity that holds it together. Don't use the Roche limit formulas in the text but estimate the break-up distance for cube of density 1000 kg m^{-3} approaching Jupiter. How close can the cube get to Jupiter before it is pulled apart? **Answer in terms of Jupiter radii.**

Assume that the comet is a cube and the only force holding the cube together is just the gravitational attraction between the two halves of the cube. It is ok to consider the mass of each cube-half to be a point mass at the center of the cube half.



2) Estimate the time (**in years**) it would take for the Yarkovsky effect to cause a 1 meter cube to spiral into the Sun from 1AU. Assume that the leading edge of the cube has a temperature of 400K and the trailing edge has a temperature of zero. In this case the (retarding) Yarkovsky force is equal to the radiated power (= hot area times σT^4) divided by the speed of light. A crude but convenient way to estimate the lifetime is to assume that the (Yarkosky force at 1AU) X (lifetime) is equal to the total momentum of the cube at 1AU [ie "impulse" $F\Delta t= d(MV)$ "change in momentum]. The cube would have a speed of 30km/s at 1AU.

3) The Poynting-Robertson effect is a caused by the drag component (the vector component against the direction of motion) of the light pressure force. As was done in the previous problem use the crude approximation that the force at 1AU times X the lifetime = the initial momentum and estimate the time it takes for a 0.1 mm black particle to spiral from 1AU to the Sun. **answer in years**

4) The Hill radius is the maximum distance from a planet that a moon can orbit with a stable orbit. The region inside this radius is called the Hill sphere and it is basically the region between the L1 and L2 Lagrangian points. Outside the Hill sphere the Sun's gravity prevents a moon from having a circular planet-centered orbit. One way to approximate the Hill radius is to consider L1, the place between a planet and the Sun where the angular rotation rate around the planet matches the angular rate of the planet around the sun. The angular rotation rate is $\omega = v/r$ for circular orbits. Derive an equation for the Hill radius in terms of the mass of the Sun, the mass of the moon and the distance from the Sun. Which planet in the solar system has the largest Hill radius and how big is it relative to the planet's orbital distance (this is a ratio)?

5). Imagine that you are an alien astronomer trying to use telescopes to detect planets around the Sun. For each of the four detection techniques, estimate the ratio of the amplitude of the signal from the Earth relative to that from Jupiter. As a lucky alien you are observing the planetary system edge on and you only have to solve this problem to one or two significant figures.

Reflex motion of the Sun. The ratio of the amplitudes of the angular wobble of the sun caused by Earth and Jupiter.

Doppler motion of the Sun. The ratio of the amplitudes of the doppler velocity of the sun caused by Earth and Jupiter.

Occultation. The ratio of the amplitudes of the dip in solar brightness when the Earth and Jupiter pass in front of the Sun.

Direct imaging. The ratio of the brightness of Earth and Jupiter in reflected sunlight.