1) A comet approaches Earth on an elliptical orbit with a perihelion of 1 AU. After a close encounter with Earth, the comet is on an orbit that has its aphelion at 1 AU and perihelion at 0.4 AU (Mercury). What was the eccentricity of the original orbit? The comet’s orbit is in the orbital plane as Earth and it orbits the Sun in the same direction as the planets. For this problem use the Earth’s speed around the Sun as 30 km/s. hint: The approach speed relative to Earth \((V_{\infty})\) on the initial orbit is the same as the exit speed relative to Earth on the smaller orbit. Initially it is traveling faster than Earth at 1 AU and after the encounter it is traveling slower.

2) Quick derivation problems:

2A) Show why an object orbiting just above the surface of a body of any size will have the same orbital period as long as the bodies being orbited have the same density.

2B) Even though the Sun is vastly more massive that the Moon, its tidal force on Earth is just a little under half that of the Moon. The Sun’s density is 1.4 and the Moon’s density is 3.4 and both the Moon and the Sun subtend about 0.5 degrees of angle in the sky. Show why the tidal force effects on a body depend on the angular size of the body causing the tide times its density.

3) The Rosetta spacecraft recently made the first measurements of the density of a comet and found that it has a density of \(~500\text{ kg m}^{-3}\). How close could this comet approach Jupiter without being pulled apart by Jupiter’s tidal force. Assume that the body is held together only by its own gravity - it has no strength. Although the comet has an interesting shape, for this problem, we are going to model it as a constant density cube facing Jupiter. Don’t just use Roche limit formula for this problem but estimate the critical breakup distance for this cubic shape.

It is ok to consider the mass of each cube-half to be a point mass at the center of the cube half. Outside the critical distance, the self gravity force between the two cube halves is greater than the differential force of Jupiters’s gravity (dF/dr times \(\Delta r\), \(\Delta r\) is the distance between the centers of mass of each cube half) that is trying to pull the cube apart. \(R_{\text{Jupiter}}= 71,500\text{ km}, M_{\text{Jupiter}}= 1.9 \times 10^{27}\text{ kg.}\)

4) Estimate the time (in years) it would take for the Yarkovsky effect to cause a 10 centimeter cube with a density of \(1000\text{ Kg m}^{-3}\) to spiral into the Sun from a location in the asteroid belt at 3 AU. The cube has close to a circular orbit and it has a leading edge (always facing in the direction of motion) that is heated to an average temperature of 400K.
The trailing edge has a temperature of zero. For the cube, the retarding Yarkovsky force is equal to the radiated power (= hot area times \( \sigma T^4 \)) divided by the speed of light. A crude but convenient way to estimate the lifetime is to assume that the (Yarkovsky force at 3AU) \( X \) (lifetime) is equal to the total momentum of the cube at 3 AU [ie “impulse” \( F\Delta t = d(MV) \) “change in momentum”].

Note: The actual thermal emission from a flat surface varies as with the cosine of the angle from the normal to the surface. This is called “Lambertian” emission. Photons emitted at an angle will also produce less drag force because only the cosine component of the force vector is in the direction of the particle motion. Fortunately for this problem with the cube, the cosine effect is a somewhat minor and you can ignore it and assume the actually unrealistic case where all emitted photons are normal to the leading edge cube face.

5) The Hill radius is the maximum distance from a planet that a moon can orbit with a stable orbit. The region inside this radius is called the Hill sphere and it is basically the region between the L1 and L2 Lagrangian points. Outside the Hill sphere, the Sun’s gravity prevents a moon from having a circular planet-centered orbit. One way to approximate the Hill radius is to consider L1, the place between a planet and the Sun where the angular rotation rate around the planet matches the angular rate of the planet around the Sun. The angular rotation rate is \( \omega = \frac{v}{r} \) for circular orbits. Derive an equation for the Hill radius in terms of the mass of the Sun, the mass of the planet and the distance from the Sun. When the planets were forming from the original disk of gas that orbited the young Sun, a massive planet could form a ring-shaped gap in the disk when the diameter of planet’s Hill sphere was equal to the thickness of the disk. Use your equation to estimate the disk thickness that could lead to the formation of ring-shaped gap by Jupiter at its final mass. \( M_{\text{Jupiter}} = 1.9 \times 10^{27} \) kg