Today

- Origin of the elements
- Orbits
- Sun
Why the Big Bang failed to make elements beyond Li

No stable mass 5 or 8 elements

$^3\text{He}$  $^4\text{He}$  $^6\text{Li}$  $^7\text{Li}$  $^8\text{Be}$  $^9\text{Be}$  $^{10}\text{B}$  $^{11}\text{B}$  $^{12}\text{C}$

$^4\text{He} + ^4\text{He} = ^8\text{Be}$

*BUT* its half-life $\sim 10^{-16}$ sec!

No $^5\text{He} + ^4\text{He}$ to make $^9\text{Be}$
In low mass stars like the Sun

if star mass \( > 0.07M_\odot \) its core temp \( T_{\text{core}} > 10^7 \) K

\( (M_\odot = \text{solar mass}) \)

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu )</td>
<td>14 x 10^9 yrs</td>
</tr>
<tr>
<td>( ^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma )</td>
<td>6 seconds</td>
</tr>
<tr>
<td>( ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2^1\text{H} )</td>
<td>10^6 yrs</td>
</tr>
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</table>

Effectively turns 4 protons into \(^4\text{He}\)

\( 4 ^1\text{H} \rightarrow ^4\text{He} + 2e^+ + 2\nu \)

0.008 of the mass “burned” goes to \( mc^2 \) energy
At $T > 10^8$ K
"He burning" - triple alpha

The secret of the stars - the pathway to heavier elements
Requires ~ three body collision $^8$Be ~ $10^{-16}$ seconds!
Triple Alpha needs:

1. Temperature $> 10^8$ K
2. Dense gas for 3-body collisions

Did not happen in the Big Bang

Because the density was too low when the T was high enough
Other fusion reactions - alpha reactions

$^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O}$
$^{16}\text{O} + ^4\text{He} \rightarrow ^{20}\text{Ne}$
$^{20}\text{Ne} + ^4\text{He} \rightarrow ^{24}\text{Mg}$
  $\rightarrow ^{28}\text{Si}$
  $\rightarrow ^{32}\text{S}$
  $\rightarrow ^{36}\text{Ar}$
  $\rightarrow ^{56}\text{Fe}$

Some of these occur in red giant stars, others in supernovae.
Hydrogen to iron
fusion reactions- exothermic
binding energy increases with mass
product is less massive than inputs
“lost mass” releases $mc^2$ energy
increasing coulomb barrier with higher charge
requires higher temperatures

Above iron
binding energy decreases with mass
reactions generally not exothermic

Main reactions involve **neutron capture**

When n capture leads to beta decay, a new element is formed

*In beta decay, a neutron is lost, a proton is gained and a beta (e⁻) is ejected*
In Stars - fusion reactions below Fe – n capture beyond

peak

NUCLEAR COOKING  NEUTRON CAPTURE

→ coulomb barrier  → no barrier just n
Two major types of neutron capture reactions

The s process
When the time between neutron captures is long
occurs in red giant cores – low n flux

The r process
When the time between neutron captures is short
occurs in A) core collapse supernova
      B) perhaps in neutron star mergers
s & r processes
s from AGB stars
r from SNell (includes 8-11, 12-25 & >25M☉)
The special case for iron

Thermonuclear detonation of a CO white dwarf (triggered by matter from companion star)

In a few seconds time, much of the CO white dwarf mass is turned into $^{56}\text{Ni}$.
$^{56}\text{Ni}$ decays to $^{56}\text{Co}$ decays to $^{56}\text{Fe}$ (which is stable)
Only the blue isotopes are stable
Number of protons

Number of neutrons

Valley of $\beta$ stability

Bi

the end of the s process

Neutron drip line

Beta decay

Strait of radioactivity

Uranium island

Beta-delayed fission

Ocean of instability

Super-heavy island
Making elements with neutrons - The S (slow) process

Low neutron flux
Time between n captures is $\gg$ decay times

<table>
<thead>
<tr>
<th>Number of protons</th>
<th>Number of Neutrons</th>
</tr>
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<tbody>
<tr>
<td>$^1A$</td>
<td>Stable</td>
</tr>
<tr>
<td>$(1+1)A$</td>
<td></td>
</tr>
<tr>
<td>$(1+2)A$</td>
<td></td>
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<tr>
<td>$(1+3)A$</td>
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<tr>
<td>$(1+4)A$</td>
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<tr>
<td>$(1+6)A$</td>
<td></td>
</tr>
<tr>
<td>$(1+5)B$</td>
<td>$(1+7)C$</td>
</tr>
<tr>
<td>$(1+6)B$</td>
<td>$(1+8)C$</td>
</tr>
<tr>
<td>$(1+10)B$</td>
<td></td>
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<tr>
<td>Element A</td>
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<tr>
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<tr>
<td>Element C</td>
<td></td>
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<tr>
<td>Element D</td>
<td></td>
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<tr>
<td>Element E</td>
<td></td>
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<tr>
<td>Element F</td>
<td></td>
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<tr>
<td>Element G</td>
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Direction of $\beta$ decay:

$n \rightarrow P^+ + e^-$ (constant mass)
The S process

Low neutron flux
Time between n captures is >> decay times

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The S process

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The diagram shows the stability of different combinations of protons and neutrons, with stable and unstable states indicated by different shading.
The S process

Number of protons

Number of Neutrons

Stable

Unstable
The S process

Number of protons

Number of Neutrons
The S process
The S process

\[ \beta \text{ decay - lose a neutron, gain a proton, emit an } e^- \]

Number of Neutrons

a new element – one more proton!
The S process

Number of protons

Number of Neutrons
The S process

Number of protons

Number of Neutrons
The S process

Another $\beta$ decay - lose a neutron, gain a proton, emit an $e^-$

Number of Neutrons

Another new element – another proton!
The S process

Number of protons

Number of Neutrons
### The S process

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<td><img src="image" alt="Diagram" /></td>
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The S process

another $\beta$ decay - lose a neutron, gain a proton, emit an e$^-$
Number of Neutrons

another new element – another proton!
The S process

Number of protons

Number of Neutrons
The S process

Number of protons

Number of Neutrons

a new element – one more proton!
The S process

Number of protons

Number of Neutrons
The S process

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The S process

Number of protons

Number of Neutrons
The S process

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The S process

Number of protons

Number of Neutrons

a new element – one more proton!
The S process
The S process

Number of protons

Number of Neutrons

* a new element – one more proton! *
The S process

Number of protons

Number of Neutrons
The S process

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The S process

The $s$ processes ends at Bi – the heaviest stable element
The R (rapid) process

High neutron flux (supernova or merging neutron stars)
Time between n captures is << decay times

Direction of β decay
\[ n \rightarrow P^+ + e^- \] (constant mass)

Number of Neutrons

Number of protons

flux of unstable neutron-rich isotopes

isotopes that cannot be made by the r process
The R (rapid) process

High neutron flux (supernova)
Time between \( n \) captures is \( \ll \) decay times

Direction of \( \beta \) decay
\[ n \rightarrow P^+ + e^- \]
(constant mass)

flux of unstable neutron-rich isotopes
The R (rapid) process

High neutron flux (supernova)
Time between n captures is $\ll$ decay times

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<th>Number of Neutrons</th>
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<tr>
<td></td>
<td>flux of unstable neutron-rich isotopes</td>
</tr>
</tbody>
</table>

Direction of $\beta$ decay
$n \rightarrow P^+ + e^-$
(constant mass) $P$

Isotopes that can only be made by the r process

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Solar Abundances ($\text{Si} = 6$)

Iron peak elements

Odd-even effect
even elements
more strongly bound
Solar Abundances

<table>
<thead>
<tr>
<th>Atom</th>
<th>Wt %</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>2.8 X 10^{10}</td>
</tr>
<tr>
<td>He</td>
<td>2.7 X 10^{9}</td>
</tr>
<tr>
<td>C</td>
<td>1.0 X 10^{7}</td>
</tr>
<tr>
<td>N</td>
<td>3.0 X 10^{6}</td>
</tr>
<tr>
<td>O</td>
<td>2.4 X 10^{7}</td>
</tr>
<tr>
<td>Ne</td>
<td>3.4 X 10^{6}</td>
</tr>
<tr>
<td>Mg</td>
<td>1.07 X 10^{6}</td>
</tr>
<tr>
<td>Si</td>
<td>1.0 X 10^{6}</td>
</tr>
<tr>
<td>S</td>
<td>5 X 10^{5}</td>
</tr>
<tr>
<td>Fe</td>
<td>9 X 10^{5}</td>
</tr>
</tbody>
</table>

CNO = 1.2 wt %

Mg Si S Fe = 0.3 wt %

MgO SiO₂ S FeO = 0.46 wt %

OXIDIZED TERRESTRIAL PLANET

<table>
<thead>
<tr>
<th>Element</th>
<th>Wt %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg</td>
<td>14 wt %</td>
</tr>
<tr>
<td>Si</td>
<td>15 wt %</td>
</tr>
<tr>
<td>S</td>
<td>9 wt %</td>
</tr>
<tr>
<td>Fe</td>
<td>26 wt %</td>
</tr>
<tr>
<td>O</td>
<td>34 wt %</td>
</tr>
<tr>
<td>Na, Al, Ca, Ni</td>
<td>percent each</td>
</tr>
</tbody>
</table>

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Galaxies are element producing machines

Heavy elements increase over time

The oldest stars contain fewer heavy elements

Heavy element abundances are determined by the history of star formation and recycling

Some elements returned into interstellar space to make new stars

Iron is delayed relative to alpha element production

Uranium and thorium production declines with time
Supernova rate vs time

Type II supernovae are massive exploding stars
Type 1a supernovae are white dwarfs in binary systems

Composition varies somewhat with time and place in the Galaxy.
Orbits
Orbit shape terminology

- **perihelion** distance: $a(1-e)$
- **aphelion** distance: $a(1+e)$

- $a = \text{semi-major axis}$
- $b = \text{semi-minor axis}$
- $e = \text{eccentricity} = F/a$

$e = F/a$
- $e = 0$ for a circle
- $e = 1$ for a parabola
- $e > 1$ for a hyperbola
red string length = ae + ae + 2a(1-e) = 2a
blue string length = red string length
so C=a  so C^2 = a^2 = b^2 + (ae)^2;  b= a(1-e^2)^{1/2}
Velocity in circular orbit

\[ F = Ma \quad (F_{\text{grav}} = \text{centripetal force}) \]

\[
G \frac{M_* M}{r^2} p = \frac{M}{p} \frac{V^2}{r}
\]

\[ V^2 = \frac{GM_*}{r} \]

\[ V = \sqrt{\frac{GM_*}{r}} \approx 30 \frac{1}{\sqrt{r}} \text{ kms}^{-1} \quad (\text{approximation for } r \text{ in AU}) \]

for orbits about the sun (or any 1M_\odot star)
Total energy in circular orbits

\[ V^2 = \frac{GM}{r} \]
\[ \parallel \parallel \]
\[ 2 \frac{KE}{\text{mass}} = - \frac{PE}{\text{mass}} \]

For circular orbit

\[ PE = -2KE \]

Total energy = KE + PE = 1/2 PE = -1/2 \ GM/r

Total energy (per unit mass) for a body in an elliptical orbit

\[ E_{total} = -\frac{GM}{2a} \]
Energy in bound and unbound orbits

\[ E_{\text{total}} = -\frac{GM}{2a} \]

\[ E_{\text{total}} \begin{cases} <0 & \text{for bound orbits} \\ >0 & \text{for unbound (hyperbolic) orbits} \end{cases} \]

\[ KE=-1/2PE \text{ for circular orbit} \]
\[ KE=-PE \text{ for } e=1 \text{ parabola} \]
\[ KE>-PE \text{ for } e>1, \text{ unbound hyperbolic orbit} \]
Kepler’s 3rd Law

\[ V^2 = \frac{GM}{r} \]

is essentially Kepler’s 3rd law

change \( r \) to \( a \)

and change \( V \) to \( \frac{2\pi a}{P} \)

where \( P \) is the orbital period

\[ \frac{4\pi^2 a^2}{P^2} = \frac{GM}{a} \]

Kepler’s 3rd law

\[ P^2 = a^3 \]

(a in AU and \( P \) in years)

for bodies orbiting the Sun
Estimating the mass of extra-solar planets

Measure $V^*$ and orbital period ($P$) of star orbiting center of mass

\[
\text{mass balance: } \quad M_* b = M_p a
\]

\[
M_p = \frac{b}{a} M_*
\]

how to get $b$ & $a$?

$a$ from Kepler's 3rd law

\[
M_* P^2 = a^3 \quad ; \quad \text{measure period } P
\]

determine $M_*$ from spectra

\[
b \quad \text{from } \quad V_* = \frac{2\pi b}{P}
\]
Escape Velocity

Escape occurs when KE>-PE
(PE/mass is 2 times the KE/mass of circular orbit)

Escape occurs when:

\[ \frac{1}{2} V^2 \geq -\frac{GM}{r} \]

\[ V_{escape} = \sqrt{\frac{2GM}{r}} = \sqrt{2} \, V_{circular \, orbit} \]

\( V_{escape} \) is the velocity to escape from distance \( r \)
Velocity in an elliptical orbit

\[
KE = E_{\text{total}} - P E
\]

\[
\frac{1}{2} V^2 = -\frac{GM}{2a} - \left( -\frac{GM}{r} \right)
\]

\[
V^2 = GM \left( \frac{2}{r} - \frac{1}{a} \right)
\]

\underline{for solar orbits}

\[
V \approx 30 \sqrt{\frac{2}{r} - \frac{1}{a}} \quad \text{km}^{-1} \quad (r \ & \ a \ \text{in AU})
\]
\[ V_p = V_{\text{escape}} = 1.414V_0 \]

- \( V_p = \) velocity at perhelion
- \( V_0 = \) velocity for circular orbit
  \[ = \left( \frac{GM}{r} \right)^{-1} \]
Sun- Basic Data

- $R_\odot \approx 100R_\oplus$; $M_\odot \approx 1000M_\text{jupiter}$
- $T_{\text{core}} \approx 15$ million K
- $T_{\text{effective}}$ (effective “surface” temperature) = $5800K$

- Primary roles: gravity to hold solar system together
  heats surfaces of SS bodies

- Other effects
  - Solar wind - mag field and spin rate decrease over time time
  - Solar constant (power density at 1 AU) - changes with time

Evolution on the main sequence
- $(T_{\text{core}}$ increases as He/H increases in core) $P=Nkt$ (t increases as n decreases for $\sim$constant P)
- $dL_\odot/dt \sim 10\%$ by$^{-1}$, factor of 2.5 as a main sequence star

Towards the end of its life the Sun loses mass
and becomes very large & bright
sunspots
cooler ~ 1000k cooler
higher B field
The $\omega$-effect
MONTHLY AVERAGE SUNSPOT NUMBER

11 yr cycle
DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)

22 year magnetic cycle.

Magnetic polarity reverses every 11 years

AVERAGE DAILY SUNSPOT AREA (% OF VISIBLE HEMISPHERE)

http://science.nasa.gov/ssd/pd/solar/images/hflv.gif

NASA/NESSIC/HATHAWAY 2004/09