Today

Oct 7

PS #1 solutions
orbital effects
tides
nongravitational effects on orbits
Exoplanets

latest comet images from Rosetta
1. Power in = Power out
\[
\frac{\text{Power in}}{\text{area}} = \frac{\text{Power out}}{\text{area}}
\]
\[
= \sigma T^4 = 5.67 \times 10^{-8} (392)^4 = 1339 \text{ watts/m}^2
\]

Total power from Sun
\[
= (1339 \text{ watts/m}^2) \times \text{surface area of AU radius sphere}
\]
\[
= (1339) 4\pi (1.5 \times 10^{11} \text{ m})^2 = 3.8 \times 10^{26} \text{ watts}
\]

1.339 \text{ watts/m}^2 \times 6 \text{ sides}

All 6 sides same temp

Power in = Power out

1339 \text{ watts/m}^2 = 6 \sigma T^4

\[
T = \left[ \frac{1339}{6} \left( 5.67 \times 10^{-8} \right) \right]^{\frac{1}{4}} = 250 \text{ K}
\]

\[
\Delta T = 392 - 250 = 142 \text{ K cooler}
\]
(2) 

Power radiated = \( \Delta T^4 \) (surface area of Sun)

Size of Sun?

\[ S = r \theta = (1.5 \times 10^6 \text{m}) \left( \frac{1 - \frac{2 \pi}{360}}{2} \right) = 1.3 \times 10^9 \text{m} \text{ (diameter)} \]

Power radiated = \( (\Delta T^4) \) (surface area)

\[ 3.8 \times 10^{26} \text{watts} = \frac{5.67 \times 10^8 \ T^4 \ \pi \left( \frac{1.3 \times 10^9}{2} \right)^2}{4} \]

\[ T = 5960 \text{ K} \]
(3) solar lifetime?

Total E from "burning" 0.1 M_0 of hydrogen

\[
(2 \times 10^{-3} \, kg)(0.1)(0.008) \, c^2 = 1.44 \times 10^{19} \, J
\]

\[
\text{Lifetime} = \frac{E_{\text{total}}}{E_{\text{radiation}}} = \frac{1.44 \times 10^{19} \, J}{3.8 \times 10^{26} \, J/s} = 3.8 \times 10^{17} \, \text{sec}
\]

\[
\text{Solar luminosity} = \frac{3.8 \times 10^{17} \, \text{sec}^{-1}}{4 \times 10^7 \, \text{sec} \, \text{yr}^{-1}} = 1.2 \times 10^{10} \, \text{yr}^{-1}
\]

\[\text{Without nuclear energy}\]

\[
\text{Grav PE} = -\frac{3}{5} \frac{Gm^2}{r} = \frac{3}{5} \times 6.67 \times 10^{-11} \frac{(2 \times 10^{29})^2}{6.96 \times 10^8}
\]

\[
= 2.3 \times 10^{41} \, J
\]

\[
\text{Lifetime} = \frac{E_{\text{total}}}{\text{dE/dt}} = \frac{2.3 \times 10^{41} \, J}{3.8 \times 10^{26} \, J/s}
\]

\[
= 6 \times 10^{14} \, \text{sec} \sim 2 \times 10^7 \, \text{yr/s}
\]

\[
\text{Mass loss rate} = 0.1 \, M_0 \, (0.008) \frac{1 \, \text{lifetime}}{(2 \times 10^{29})(0.008)} \frac{(1.2 \times 10^{10})}{4 \times 10^9} \, \text{yr}^{-1}
\]

\[
\text{[also } L_0 = \frac{m \, c^2}{\text{time}}]\]
⑦ Earth-Moon distance = 384,000 km
Alpha Centauri distance = 4.4 x 10^16 m

\[ \theta = \frac{3.8 \times 10^5}{4.4 \times 10^{16}} = 8.6 \times 10^{-9} \text{ radians} \]

\[ \frac{r}{D} = \frac{5 \times 10^{-7}}{8.6 \times 10^{-9}} = \frac{58}{3} \]
\[ \Delta T = \frac{2 R_E}{c} = \frac{2 \left(6.4 \times 10^6 \text{m}\right)}{3 \times 10^8 \text{m/s}} = 0.045 \text{sec} \]

speed at equator = \[ \frac{2\pi R_E}{\Delta y} = \frac{2\pi \left(6.4 \times 10^6 \text{m}\right)}{8.6 \times 10^4} = 467 \text{m/s} \]

\[ \frac{\Delta F}{F} = \frac{467}{c} = \frac{467}{3 \times 10^8} = 1.6 \times 10^{-6} \]

\[ \Delta F = \left(1.6 \times 10^{-8}\right) \left(10^9 \text{Hz}\right) = 1.6 \times 10^3 \text{Hz} \]

full width H = 2 \Delta F = 3200 \text{Hz}
A Few More Slides on the Sun
Charged particle acceleration at magnetic reconnection sites

Reconnection site

Energy outflows

Turbulence acceleration region, Coronal X-ray emission

Looptop source

Escaping particles

Thick-target footpoints
Charged particle outflow from the sun

Range from high energies (solar flare particles – to >Gev) to the solar wind ( ~ 1 kev)

Solar wind

$V \sim 350 \text{ km/s (4 days to reach Earth)}$
~ solar composition
Density at 1 AU a few cm$^{-3}$

Measuring solar wind speed with a comet tail
solar corona
$10^6$ K low density gas
seen in an eclipse
9 billion years of nucleosynthesis summary

He to Fe  (the alpha elements)
  fusion reactions – red giant stars & core collapse SN

Fe peak elements
  Type 1A SN (detonation of CO white dwarf star)

Beyond Fe
  neutron capture  r & s process
  red giant stars & core collapse SN
Speed in a circular orbit

\[ V = \sqrt{\frac{GM}{r}} \quad \text{and} \quad V_{\text{escape}} = \sqrt{\frac{2GM}{r}} \]

Total energy

\[ E_{\text{total}} = -\frac{GM}{2a} \]

Speed in an elliptical orbit

\[ V^2 = GM \left( \frac{2}{r} - \frac{1}{a} \right) \]

Impact speed

\[ V_{\text{impact}}^2 = V_\infty^2 + V_{\text{esc}}^2 \]
Estimating the mass of extra-solar planets

Measure $V_*$ and orbital period ($P$) of star orbiting the center of mass

$V_* \rightarrow b \rightarrow a \rightarrow \text{Center of mass of star & planet}$

mass balance: $M_\ast b = M_p a$

$M_p = \frac{b}{a} M_\ast$

how to get $b$ & $a$?

$a$ from Kepler's 3rd law

$M_\ast P^2 = a^3$ ; measure period $P$

determine $M_\ast$ from spectra

$b$ from $V_* = \frac{2\pi b}{P}$

star radial velocity curve modulated by unseen planet
Speed in an elliptical orbit

\[
KE = E_{\text{total}} - PE
\]

\[
\frac{1}{2} V^2 = - \frac{GM}{2a} - \left( - \frac{GM}{r} \right)
\]

\[
V^2 = GM \left( \frac{2}{r} - \frac{1}{a} \right)
\]

for solar orbits

\[
V \approx 30 \sqrt{\frac{2}{r} - \frac{1}{a}} \text{ km s}^{-1} \text{ (r & a in AU)}
\]

This gives speed not velocity (no direction)
Escape

Escape occurs when KE > -PE  (KE + PE > 0)
(parabolic or hyperbolic orbit)

KE is 2 times the KE/mass of circular orbit
When:

\[
\frac{1}{2} V^2 \geq - \frac{GM}{r}
\]

\[
V_{\text{escape}} = \sqrt{\frac{2GM}{r}} = \sqrt{2} \ V_{\text{circular orbit}}
\]

\( V_{\text{escape}} \) is the velocity to escape from distance \( r \)
Falling into or climbing out of a potential well
add energies - (proportional to \( V^2 \)) not velocities

\[
V_\infty^2 = V_{\text{launch}}^2 - V_{\text{esc}}^2
\]

\( \text{(KE}_\infty = \text{KE}_{\text{launch}} - E_{\text{escape}}) \)

\[
V_{\text{impact}}^2 = V_\infty^2 + V_{\text{esc}}^2
\]

\( \text{(KE}_{\text{impact}} = \text{KE}_\infty + E_{\text{escape}}) \)
Minimum energy orbit to get from orbit A to B

Also called a Hohmann transfer ellipse

Smallest orbit smallest a to get from A to B lowest energy

launch time “window”
Aerobraking - using atmospheric friction to circularize an orbit around a planet

Force in the direction of motion
At perigee causes big change at apogee

atmospheric drag lowers apogee

small thrusts at Apogee lift perigee
Gravity assist
by the Voyager spacecraft
Gravity assist - 3 body interaction - comet-Jupiter-Sun

ORBITAL EVOLUTION OF COMET WILD 2

ORBIT OF URANUS

ORBIT OF WILD 2 PRIOR TO 1974 CLOSE APPROACH TO JUPITER

ORBIT OF JUPITER

ORBIT OF SATURN

EARTH'S ORBIT

ORBIT OF WILD 2 SUBSEQUENT TO CLOSE APPROACH TO JUPITER ON SEPT. 10, 1974 (MIN. SEPARATION = 0.006)

Jupiter
hyperbolic relative to Earth

\[ V_{\text{E}} = 30.2 \text{ km/s} \]
\[ V_{\infty} = 8.9 \text{ km/s} \]
\[ V_{\text{S/C IN}} = 30.1 \text{ km/s} \]
\[ V_{\text{S/C OUT}} = 35.3 \text{ km/s} \]
Resonance effects in the asteroid belt

When the orbital period is a simple fraction of that of another body

Kirkwood gaps when \( \frac{P_{\text{asteroid}}}{P_{\text{jupiter}}} \) is a simple fraction

stable resonances
unstable resonances
Unstable resonances in the asteroid main belt
The Kirkwood Gaps

Mean Motion Resonance
(Asteroid: Jupiter)

Number of Asteroids
(per 0.0005 AU bin)

Semi-major Axis (AU)
A stable resonance – the Hilda asteroids orbit the sun 3 times when Jupiter orbits 2 times (close to Jupiter’s orbit but never close to Jupiter)
Mimas →

2:1 resonance in Saturn’s rings

Huygen’s gap
Interesting effects among orbiting bodies

The red ball pushes the blue ball outwards to a higher energy orbit

It pushes the green ball inwards to a lower energy orbit
moons can make nearby rings & gaps

Sheparding moons

Diagram showing trajectories of moons and particles in a ring system.
How Pan Creates the Encke Gap

How Pandora and Prometheus Shepherd the F-Ring
Saturn’s Encke division caused by Pan - at 35 km moon
co-orbital satellites (rare)

higher energy

lower energy orbit
Horseshoe orbit

gains energy and falls behind

overtakes planet

Tadpole orbit

reference frame rotating with planet
Lagrangian points in the restricted 3 body problem
Tides

effects of the differential force of gravity

force relative to center
(force - center force vector)

result

BIG MASS

\[ F \propto \frac{1}{r^2} \]

\[ \frac{dF}{dr} \propto \frac{1}{r^3} \]
A few tidal effects
Tidal breakup - Roche's limit

Self Gravity vs tidal force

Breakup of Comet Shoemaker-Levy 9 from close pass to Jupiter
Tidal locking
Spin period = orbit period

Planets in the habitable Zones of low mass stars are probably tidally locked
Tidal locking
Spin period = orbit period

Planets in the habitable Zones of low mass stars are probably tidally locked
Tidal locking
Spin period = orbit period

restoring torque

Planets in the habitable Zones of low mass stars are probably tidally locked
Tidal locking
Spin period = orbit period

Planets in the habitable Zones of low mass stars are probably tidally locked
Tidal locking complicates life on planets near stars.
Mercury: Orbit period = 3/2 spin period

Hg has a very eccentric orbit $e = 0.2$
aphelion/perihelion = 1.5

1.5 spins

Mass moment lined up with Sun at perihelion

full spin

half spin
Tidal effects – cause the moon to move outward & the Earth to spin faster

$F_l$ force from leading tidal bulge closer to Moon

If the moon had a retrograde orbit tidal effects would cause its orbit to decay!

$F_l > F_t$ and has a net component in the direction of the Moon’s motion
Adds energy to Moon causing it to spiral out
Tidal heating of satellites & planets

\[ \frac{dF}{dr} \propto r^{-3} \quad \text{much stronger at perijove} \]

100 m tides on Io
Io as seen from the New Horizons spacecraft