Today Nov 4
quiz solutions

accretion
giant planet formation
migration
quiz scores

average -122

top score 187
1. Describe how atmospheric distortion can be nearly eliminated for ground based telescopes and what determines the ultimate angular resolution of a telescope?

Detect wavefront distortion
Correct with deformable mirror

\[
\text{resolution} = \frac{\lambda}{D} \quad \text{photon wavelength}
\]

\[ \text{mirror diameter} \]

2. Use a sketch to show how tidal effects might cause the Earth to impact the Sun when the red giant Sun is nearly as large as the Earth’s orbit? Also describe how tidal effects can circularize the orbit (decrease its aphelion) of a “hot-Jupiter” that initially had an elliptical orbit and how they can, in special cases, cause a moon to spiral inwards and impact the planet that it is orbiting.

Earth

\[ \text{Tidal bulge} \]

\[ \text{Moon in retrograde orbit} \]

\[ \text{Loss of energy (due to drag)} \]

\[ \text{Lowerers aphelion distance} \]
3. A transiting planet is observed orbiting a 1 solar mass star. What observations and calculations have to be done to determine the planet's density? How can it be determined if the planet is in the star's habitable zone? What happens to the transit light curve if the planet's path goes in front of a large star spot?

\[ \text{radius from transit depth} = \left( \frac{R_p}{R_*} \right)^2 \]
\[ \text{mass from radial velocity - Kepler's 3rd law mass formula} \]
\[ 2 \pi R \text{ in H}_2 \text{? need orbital radius (r)} \]
\[ m_p^2 = a^3 \text{ Kepler's 3rd law} \]

Transit light curve

\[ \text{Time \rightarrow brighter when over a star spot} \]

4. What mass criteria are traditionally used to distinguish stars from brown dwarfs and from planets and what are they based on? Why are some exoplanets much larger than expectations that were based on the properties of solar system planets?

Normal star \( M > 0.08M_\odot \) \(( \sim 80 M_\oplus) \)
\[ T_{\text{core high enough to "burn" H \rightarrow He}} \]

Brown dwarf \( M > 0.01 \) \(( \sim 13 M_\oplus) \)
\[ T_{\text{core high enough to "burn" deuterium}} \]

Jupiter was expected to be nearly the largest planet

"Puffed up" Jupiters are larger because they aren't still expected
5. What are the important ways that exoplanets and exoplanet systems differ from what we have in the solar system?

- Hot Jupiters
- Few Jupiters
- Few other planets when there is a hot Jupiter
- Super Earths
- Mini Neptunes
- "Packed" systems
- Typical planets close to star - and small few $\frac{P}{E}$
- Binary stars
- Hi $\sim$ retrograde orbits

6. A black sheet of paper is ejected from a comet and it always faces the Sun. How thin must the paper be to have the force of sunlight equal to pull of the Sun's gravity? If the comet was on an elliptical orbit where the kinetic energy is just 1% less than the potential energy, how much thicker could the paper be and still be ejected from the solar system. The paper's density is $\rho$.

\[ \text{Force balance} \]

\[ \frac{L_0}{4\pi r^2} \left( \frac{1}{c} \right) \text{area} = \frac{(\text{area})(T) \rho G M_0}{r^2} \]

- If $KE = -0.99 PE$
  - Only a 1% change in "effective PE" makes $KE > PE$
  - The paper can be 100 times thicker $\sim 100,000 \text{ cm}$ and still be "blown out"
7. What determines the inner and outer limits of a star's habitable zone (HZ)? What properties of a planet might extend the traditional inner and outer limits of the HZ? What process complicates habitability of planets in the HZ of low mass stars that is not important for planets orbiting in the HZ of stars similar to or more massive than the Sun?

The power density from the star $\frac{L_*}{4\pi r^2}$
inner edge - too hot - water lost by moist or runaway greenhouse
outer edge - too cold - water freezes - CO$_2$ and dry ice
Dry planets (Dune planets) might exist closer to star
Thick hydrogen atmospheres might allow planets to keep oceans beyond the "outer limit-HZ"

HZ for low mass stars is within the "Tidal Lock" zone - spin once per orbit.

8. How can it be determined if a planet is orbiting a distant star in either a prograde or retrograde direction relative to the star's spin direction. How could a planet get into a highly inclined or retrograde orbit if it formed in a circular prograde orbit?

Doppler shift of star during transit

[Diagram of a planet and star with labels indicating Doppler shift: red shift, blue shift.]
9. Show examples of orbital resonances with Jupiter that allow some asteroid orbits to be stable in otherwise hostile locations and also show examples of resonances that make some asteroid orbits unstable.

Stable - Trojans \( P = P_J \)

Unstable - Kirkwood Gaps
\[
P = \frac{1}{2} P_J, \frac{1}{3} P_J, \frac{2}{5} P_J
\]

10. Derive the equation for the aphelion speed of a solar system body in an orbit with a semimajor axis \( a \) and eccentricity \( e \) ?

\[
V^2 = \frac{GM}{r} = \frac{GM}{a(1+e)} = \frac{GM}{a} \left( \frac{1+e}{1+e} \right)
\]

11. Sometimes transiting planets dim the star earlier or later than predictions based on earlier observations of transits. What causes this and why is this useful?

Transit timing variations (TTV)
caused by orbital period changes of the planets

Implies the presence of other planets
Sometimes provides estimates of planet masses
Can detect planets that are not aligned for transits!
12. Only two planets form around a star. They have identical masses and miraculously they managed to form in circular orbits the same distance (10 AU) from the star. After many mutual interactions that changed their orbits, one is just barely ejected on an escape orbit from the star. What are two orbits that the second planet could have?

\[ E_{\text{tot}} (\text{2 planets}) = -2 \frac{GM}{r_1} + \frac{GM}{r_2} \]

When planet is lost - its \( E \) goes to \( -\infty \).

So the lost planet has all the \( E = \frac{GM}{r_2} \).

\[ \frac{GM}{r_2} = \frac{GM}{24.15 \text{km}} \]

at \( \text{10AU} \rightarrow \frac{1}{2} \)

13. If the 10 km asteroid that killed the dinosaurs hit Earth at 15 km/s, how fast was it traveling before it was accelerated by Earth’s gravity? If was on an elliptical orbit that ranged from 3AU to 1AU, would it have hit Earth from behind or be overtaken by Earth? (\( V_{\text{esc}} \) from Earth=11.2 km s\(^{-1}\))

\[ 15^2 = 11.2^2 + V_0^2 \]

\[ V_1 \text{AU} > 301 \text{km/s} \]

14. Why is iron much more abundant in the Sun than all of the other elements heavier than Silicon? Why it likely that Earth-like planets that formed around the very oldest stars in the Galaxy will have less iron relative to silicon and magnesium than the Earth has? How does beta decay produce elements heavier than iron and what is the difference between the \( s \) and \( r \) processes?

\[ \text{Fe at Peak of E\'s Curve} \]

\[ \text{most tightly bound/mass} \]

\[ \text{end of fusion reactions} \]

Major fraction made by Type Ia Supernovae evolute stars dumps mass on \( c \) no white dwarf \( \Rightarrow \) explosion

\[ \text{Take } t \text{ to evolve this stars} \]

\[ N \rightarrow P \]

\[ S \text{ process - slow time} \]

\[ \beta \text{ decay time} \] between \( \text{capture}\) and \( \beta \text{ decay time} \)

\[ r \text{ process - fast capture times} \ll \beta \text{ decay times} \]
15. How can moons in Saturn’s ring open up a gap in the rings and sometimes shepherd particles into a thin ring?

- gets energy from moon goes to larger orbit
- gives energy to moon goes to smaller orbit
- moves inward particle inward
- moves inward particle inward (lower energy)
- shepherd moons

16. What generally defines the inner and outer edges of the Habitable zone?
17. Jupiter is 1000 solar radii from the Sun and its diameter is 10% of the diameter of the Sun. If it was at 0.05 AU instead of ~5 AU from the Sun, how much fainter would it be than the Sun (at its maximum observed brightness) as viewed by a distant alien astronomer. What would be the probability that the observer could see it transit the Sun?

\[ 0.05 \text{ AU} = 10 R_0 \quad \frac{B_5}{B_0} = \frac{\sqrt{\frac{\text{distance}}{10^2}}}{10^2} = 10^{-4} \]

Transit probability

\[ = \left( \frac{\text{Planet radius}}{\text{Star radius}} \right)^{-1} = \left( \frac{10^{-4}}{1} \right) = 0.1 \]

18. Why do boulders orbiting in the original disk of gas that orbited the Sun, experience a headwind and how can this general process cause boulders and pebbles to both spiral into the Sun and sometimes to clump into dense concentrations?

Gas orbits slower because of the outward pressure gradient force – causes a headwind on boulders

Gas orbits at sub keplerian speed

Gas drag force moves boulders inward

Gas orbits at > keplerian speed

Gas drag boulders outwards

"Sear wind" not headwind

19. What evolutionary sequence do the following spectra of young 1 solar mass stars indicate and roughly how long does it take to go from class zero to III?

Dust around young star emits "IR excess"

Dissipates in a few million years

Less IR excess from dust

class 0 → III

(T Tauri star)
20. Derive the total energy/unit mass of a body in a circular orbit in terms of the central mass and the orbit radius. How much energy must be added to make the orbiting body escape?

\[ E_T = KE + PE = \frac{1}{2} v^2 - \frac{GM}{r} = \frac{1}{2} \left( \frac{GM}{r} \right) - \frac{GM}{r} \]

\[ = - \frac{GM}{r} \]

To escape \( KE = - PE \)

Double \( E_T \) for circular orbit.

21. Short answers (20ts total)

What single nuclear reaction makes solar mass stars last for billions of years?

\[ ^1H + ^1H \rightarrow ^2H \]

A particle is spiraling in towards the Sun on a relative circular orbit due the to Poynting-Robertson drag force from light pressure. How much higher is the force at 0.01 AU than it was at 1AU?

\[
PR_{force} \propto \frac{\text{Flux}}{V_0} \frac{1}{V_0} \frac{1}{V} \]

\[ PR_{force at 0.01AU} > 1000 \text{ higher} \]

A body is located at the L1 (Lagrangian point 1) between Earth the Sun and you can consider that it orbits the Sun and Earth once a year. Does the Sun or Earth pull harder on the body or are the forces the same?

\[ F_{sun} = F_{Earth} + \frac{v^2}{r} \]

\[ F_{sun} > F_{Earth} \]

What changes can the A) the Poynting-Roberston effect and the B) the Yarkovsky effect have on particles orbiting a planet?

\[ \text{Poynting-Roberston effect:} \quad \text{Particles spiral towards planet} \]

\[ \text{Yarkovsky effect:} \quad \text{Particles cancel out on opposite sides of planet} \]

What are the four most abundant elements in the Earth?

\[ O, \text{Mg, Si, Fe} \]
PS #5

1. \[ \frac{3}{5} \frac{G M_0^2}{r} = \frac{2}{5} \left( \frac{3}{2} \right) \frac{1}{T} \frac{M_0}{\mu M_+} \]

   \[ T = \frac{1}{5} \frac{G}{r} \frac{1}{k} \mu M_H M_0 \]

   \[ = \frac{1}{5} \frac{6.67 \times 10^{-11}}{(10^9) / (1.5 \times 10^{15})} \frac{(2 \times 1.67 \times 10^{-27})}{1.38 \times 10^{-23}} \frac{2 \times 10^3}{2} \]

   \[ T = \frac{4.3 k}{2} \]

2. \[ P^2 = a^3 = \left( \frac{10^4}{2} \right)^3 \]

   \[ P = 350,000 \text{ yrs} \]

   \[ \text{Free fall time} = \frac{P}{2} \approx 180,000 \text{ yrs} \]
Full form of Kepler's 3rd law

\[ \frac{G}{4\pi^2} m p^2 = a^3 \]
\[ m = \frac{4}{3} \pi a^3 \rho \]

\[ \frac{G}{4\pi^2} \left( \frac{4}{3} \pi a^3 \rho \right) p^2 = \left( \frac{a}{2} \right)^3 \]

\[ p^2 = \frac{(4/3) \pi}{(4/8) G \rho} = \frac{3\pi}{8G\rho} \]

Free fall time = \( \frac{p}{2} = \sqrt{\frac{3\pi}{32G\rho}} \)
Angular momentum \( L = \text{MVR} \)

\[
V^2 = \frac{GM}{r}
\]

\[
\frac{L}{M} = VR = \sqrt{\frac{GM_0}{r}} \quad r = \sqrt{\frac{GM_0}{r}} = 10^{17}
\]

\[
\frac{GM}{GM_0} = \frac{10^{34}}{(6.67 \times 10^{-11})(2 \times 10^{20})} = 7.5 \times 10^3 \text{ m}
\]

\[
= \frac{7.5 \times 10^3 \text{ m}}{1.5 \times 10^{16} \text{ m/au}} = 500 \text{ AU}
\]

\[
10 \text{ ev} = \frac{GM_0 M_p}{2q} + \text{ escape (dissolved energy)}
\]

\[
= \left( \frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{1.67 \times 10^{-29}} \right) = 1.6 \times 10^{-18} \text{ joules}
\]

\[
a = \frac{1.1 \times 10^{-7}}{1.6 \times 10^{-18}} = 6.9 \times 10^4 \text{ m}
\]

\[
= 0.46 \text{ AU}
\]
Accretion - how dust forms boulders & larger objects

\[ V = \text{average relative speed} \]

\[ \rho \text{ (kgm}^{-3}\text{)} \]

\[ \Delta m = \pi s^2 \rho V \Delta t \]

\[ \frac{dm}{dt} = \pi s^2 \rho V \] \(\text{(mass swept up)}\)

\[ \frac{dm}{dt} \propto m^{2/3} \rho V \]

\[ \frac{dm}{dt} \text{ per area is constant with constant } \rho \text{ & } V \]

\[ \frac{ds}{dt} \text{ is also constant } \{\text{like a spray can}\} \]
Gravitational enhancement during accretion

Enhancement factor \( = \frac{R^2}{r^2} \)

\( VR = V_I r \quad (\text{conservation of angular momentum}) \)

\( V_I^2 = V^2 + V_{esc}^2 \quad (\text{adding energies}) \)

Enhancement factor \( = \frac{R^2}{r^2} = \frac{V_I^2}{V^2} = 1 + \frac{V_{esc}^2}{V^2} \)

\( V_{esc} = \sqrt{\frac{2GM}{r}} \propto r \sqrt{\text{density of body}} \quad \{r = \text{radius of body}\} \)
\[ \frac{dm}{dt} = \pi s^2 \rho V \quad \text{no gravitational enhancement} \]

No gravitational enhancement

\[ \frac{dm}{dt} = \pi s^2 \rho V \left(1 + \frac{V_{esc}^2}{V^2}\right) \quad \text{with gravitational enhancement} \]

With gravitational enhancement

\[ \text{can be large if } V << V_{esc} \]
Most mass in small bodies
Runaway growth - $V$ not ruled by largest bodies

if $V$ is constant but $V^2_{\text{esc}}$ increases - $\left(1 + \frac{V^2_{\text{esc}}}{V^2}\right)$ becomes large!

$$V_{\text{esc}} = \sqrt{2GM/s} \propto s \propto M^{1/3}$$

then $\dot{M} \propto m^{2/3} \left(m^{1/3}\right)^2 = m^{4/3}$ \Rightarrow \text{runaway growth!}$

Most mass in large bodies
Oligarchic growth-- ruled by largest bodies

if $\frac{V^2_{\text{esc}}}{V^2}$ is constant

then $\dot{M} \propto \pi s^2 \propto m^{2/3}$

Planetismals are “self stirred”
The style of accretion varies over time

**Early- Runaway**
When most mass is still in small bodies
massive bodies grow at a very fast rate $V_{\text{esc}}/V$ becomes large
more massive bodies grow faster
dynamical friction and energy dissipation keeps $V_{\text{esc}}/V$
(equipartition of energy – like a gas)
$V_{\text{esc}}$ grows with time as bodies get larger

**Later- Ordered of Oligarchic growth (controlled by largest bodies)**
When most mass in in large bodies
massive bodies “stir up” orbits and $V_{\text{esc}}/V$ remains constant

**Even later- Accretion slows down when accreting bodies**
deplete their neighborhoods
When they reach their “isolation mass”

**Final assembly occurs by collision of very large bodies**
(called embryos Moon to Mars size bodies)
Transforming $V$ and $\rho$ to $\omega$ and $\Sigma$

**Surface density $\Sigma$ and volume density $\rho$**

$$H = \text{half height of the nebula at distance } r$$

$$\Sigma = \text{Surface density (mass per unit area "above and below")}$$

$$\Sigma = 2\rho H$$

so \[ \rho = \frac{\Sigma}{2H} \]

Where $\rho$ is the volume density (mass per unit volume)
V is the typical impact speeds of particles

How to relate V to the thickness of the nebula (H) and angular rotation rate (ω)?

\[ V_{\text{cir}} = \omega r \]

\{circular orbit velocity\}

\[
\frac{H}{r} = \frac{V}{\omega r}
\]

\[
H = \frac{V}{\omega}
\]

so \[ V = \omega H \]

\( H = \text{half – thickness of disk} \)

\( V = \text{typical relative velocity} \)

\( \omega = \text{angular rotation rate} \)
Transforming $V$ and $\rho$ to $\omega$ and $\Sigma$

\[ \rho = \frac{\Sigma}{2H} \]

\[ V = \omega H \]

\[ \frac{dm}{dt} = \pi s^2 \rho V \left(1 + \frac{V^2}{V^2_{esc}}\right) \]

\[ = \pi s^2 \frac{\Sigma}{2H} \omega H \left(1 + \frac{V^2}{V^2_{esc}}\right) \]

\[ = \frac{1}{2} \pi s^2 \Sigma \omega \left(1 + \frac{V^2}{V^2_{esc}}\right) \]
\[
\frac{dm}{dt} = \frac{1}{2} \pi s^2 \sum \omega \left(1 + \frac{V_{\text{esc}}^2}{V^2}\right)
\]

\[
m = \frac{4}{3} \pi s^3 \rho_p \quad \text{where } \rho_p \text{ is the density of growing body}
\]

\[
\frac{dm}{dt} = 4\pi^2 \rho_p s^2 \frac{ds}{dt}
\]

so
\[
\frac{ds}{dt} = \frac{\sum \omega}{8 \rho_p} \left(1 + \frac{V_{\text{esc}}^2}{V^2}\right)
\]

Growth rate only depends on surface density and distance from Sun if \(V_{\text{esc}}/V\) is constant.
How $ds/dt$, $\Sigma$ & $\omega$ vary with solar distance 
(assuming $V_{esc}/V$ is constant)

circular orbit velocity = $\sqrt{GM/r} = \omega r$

so $\omega = \sqrt{GM/r^3}$; $\omega \propto r^{-3/2}$

evidence from the planets suggests that $\Sigma \propto r^{-3/2}$

so $\Sigma \omega \propto 1/r^3$

Unless $V$ was much smaller than $V_{esc}$

accretion was very slow in the outer solar system
Gas – dust disks are short lived!

% disks with near IR excess

Disk frequency (%) vs Age (Myr)

Wyatt 2008
The snow line?

Distance from protoSun (AU)

- Water condensation point
- Local increase of density at Jupiter's orbit

~0.5 AU
Isolation mass

A body becomes isolated (its growth rate slows down) after it has accreted most of the mass of material in the blue annulus width = Hill sphere diameter mass = surface density $\times$ annulus area

\[ R_H = r \left( \frac{M_p}{P_{\text{star}}} \right)^{1/3} \]
Isolation Mass & the Hill Sphere

sphere of gravitational influence of body:

\[ r_{\text{Hill}} = \left( \frac{M}{3M_\ast} \right)^{1/3} r \]

Large planetesimal gobbles up most material in its annular zone on the order of Hill sphere width
The final stages of accretion involves impacts of very large bodies - due to run away growth phase.

accretion forms embryos—moon to mars-size projectiles
How water reaches Habitable Zone planets

Planet accretion simulations
Water carriers scattered from more distant regions

Sean Raymond
Terrestrial Planet compositions

Raymond et al 2013/14 PPVI

Earth’s ocean = 2 x 10^{-4} M_E
The solar system is “packed” – almost all real estate is taken.

Dynamical lifetimes of bodies in outer SS
Holman & Wisdom AJ 1993

ALL ECOLOGICAL NITCHES FILLED OUT TO 40AU
Did planet migration radically change the solar system?

The “Nice Model” - a hypothesis

Outward migration of Saturn may have dramatically destabilized the solar system
- AFTER THE GAS WAS GONE -

Many exoplanets have eccentric orbits
- evidence for past orbital instability
Outward migration of outer planets
(after the gas-disk was gone)

Due to the presence of the massive planet Jupiter

Scattering of bodies by the outer planets was unbalanced

They scattered more bodies into smaller (lower energy) orbits than into larger (higher energy) orbits

Conserving energy—this caused the outer planets to migrate outward

And Jupiter to migrate inwards
Plutinos - strong evidence for planet migration

Plutinos - bodies with 3/2 orbital resonance with Neptune orbit twice when Neptune orbits 3 times

Neptune

Pluto is located ~here
Migration of Neptune & the origin of the Plutinos

Outward migration of Neptune sweeps bodies into resonance orbits

Renu Malholtra
Outward migration of outer planets – by scattering

When a body is deflected by an encounter with Neptune, it is deflected to either a smaller or larger orbit. To conserve energy, Neptune goes into a larger (greater energy) or smaller orbit (smaller energy).

Neptune migrates outwards because it deflects more bodies inwards - towards Jupiter.
The SS became unstable 3.9 billion years ago

- Giant planets start with 5-15AU circular orbits
- $35M_\oplus$ of planetesimals beyond planets
- $P_{\text{Sat}} < 2P_{\text{Jup}}$ (=2.5 at present)
- Slow evolution of periods until $P_{\text{sat}} = 2P_{\text{Jup}}$
- Outward migration of Saturn is due to scattering bodies (similar to outward migration of Neptune)
- Resonance increases eccentricity of Jup & Saturn
- Perturbations drive SS nuts - chaos