ASTR 323 Spring 2009

Solutions to Problem Set 4 (24 points total)

Due: April 28, 2009

(CO = Carroll & Ostlie)

1. (3 pt) CO 25.14

   Solution: The radial equation is $\rho = R - R_m = A_R \sin \kappa t$ (25.34). At the midpoint of the excursion, $R = R_m$, so $\kappa t = 0$ or π. At this point the velocity is $\dot{\rho} = A_R \kappa \cos \kappa t = \pm A_R \kappa$ depending on whether κt is 0 or π. The value for $\kappa = 36.7$ km s^{-1} kpc^{-1} at the Solar circle is given just after equation (25.38), or can be computed from B and A given in Chapter 24. Now, $\dot{\rho} = u_\odot = -10$ km s^{-1} as given in equation (24.33). Since this is negative, we know that κt is π. Thus, $A_R = -10$ km s^{-1}/(-36.7 km s^{-1} kpc^{-1}) = 272 pc. This represents a lower limit on amplitude of radial oscillation since if the Sun were not at the midpoint then it would be moving slower than it is moving at the midpoint (it would either be accelerating towards the midpoint, or decelerating from the midpoint). Thus, if we used a higher velocity at the midpoint, we would get a larger excursion. A more precise estimate of the radial excursion can be obtained by using the equation for the azimuthal amplitude of oscillation (equation 25.36), and the deviation of the Solar motion from the LSR ($v_\odot$) to get the phase of oscillation for the Sun in the radial direction.

2. (3 pt) CO 25.15

   Solution: (a) (2 pt) Based on the definition of κ^2, eqn (25.29), with $\Phi_{eff} = \Phi + J_z^2/(2R^2)$, $\kappa^2 = \frac{\partial^2}{\partial R^2}\Phi|_{R_0} + 3J_z^2/R^4|_{R_0}$ since J_z is approximately constant. Now the angular momentum per unit mass is $J_z = R_0\Theta_0$. For a circular orbit at any radius (in the disk mid-plane), acceleration balances the radial gravitational force so $\ddot{R} = -\partial\Phi/\partial R + J_z^2/R^3 = 0$, or $\partial\Phi/\partial R = J_z^2/R^3 = \Theta^2/R$. So,

$$\begin{aligned} \kappa^2 &= \frac{\partial}{\partial R}[\Theta^2/R]_{R_0} + 3\Theta_0^2/R_0^2 \\ &= \left[2\frac{\Theta}{R}\frac{\partial\Theta}{\partial R} - \frac{\Theta^2}{R^2}\right]_{R_0} + 3\frac{\Theta_0^2}{R_0^2} \end{aligned}$$

$$= \quad 2\frac{\Theta_0}{R_0}\left[\frac{\Theta_0}{R_0} + \frac{\partial\Theta}{\partial R}_{R_0}\right]. \tag{1}$$

(b) (1 pt) Comparing (25.37) to equations (24.39), (24.40), and (24.43), the portion in brackets is $-2B$, while the portion outside brackets is $2\Theta_0/R_0 = 2\Omega = 2(A - B)$, so $\kappa^2 = -4B(A - B)$.

3. (11 pt) Do the first four exercises (Tidal Tails, Merger Dynamics, Real Galaxies, and Galaxy Evolution) found at

   `http://burro.cwru.edu/JavaLab/GalCrashWeb/labIntro.html`

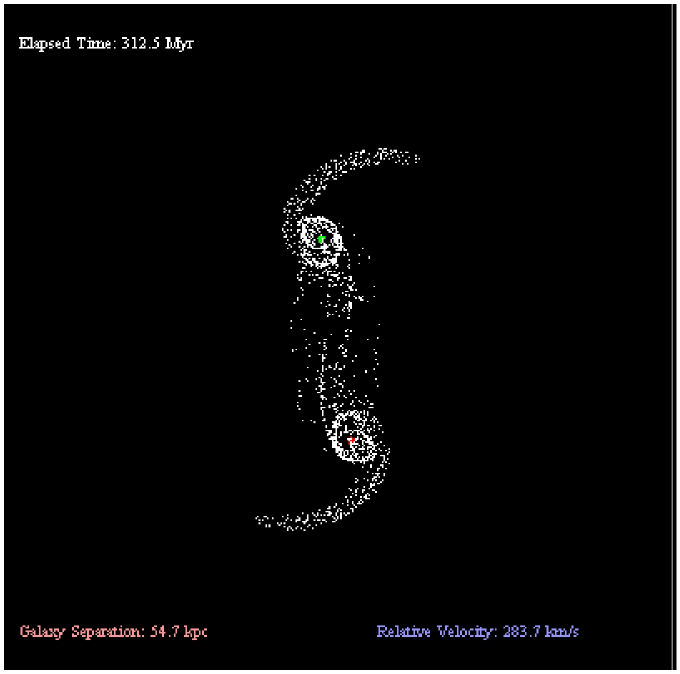
   (make sure you hit start, stop & reset before running to initialize; every time you change parameters hit reset before running again). You should read the "Background" material on how the simulations are run and information about the applet under "Controls" (both in the upper left).

   For the Real Galaxies problem, report *all* the parameters of the simulation & time at which your simulation looks like the observed image. If you can include a screen shot, even better. Note: I had to restart the applet in order to get the angles to reset.

   Solution: (a) (3 pt) Tidal Tails: This figure shows the default configuration with Friction turned off at 312.5 Myr. The tidal tails are strong for both galaxies. When the red galaxy is decreased in mass by a factor of 2, then the green galaxy has weaker tidal tails - this is because the red galaxy exerts a weaker tidal force. When the red galaxy mass is doubled, the green galaxy has stronger tidal tails.

   When the pericenter is smaller, the tidal tails become thicker, while when the pericenter is larger the tails become more narrow and shorter. Again, this is because the tidal force becomes weaker as the separation grows.

   When theta for the green galaxy is flipped by 180°, the red galaxy has a similar tidal tail, while the green galaxy has no tidal tail - it is just slightly disturbed. This is due to the fact that when the galaxies rotate in the same direction as their angular momentum, the stars closest/furthest feel a pull towards the other galaxy that is always in



the same direction, causing a stronger tidal force since it acts over a longer time. When the galaxy is counter-rotating, the stars move past one another more quickly, so the tidal force doesn't act in the same direction in the rotating frame of the stars, so the net effect is weaker. This is known as a resonance.

I find thin tidal tails when the galaxies just graze one another. For example, in the default simulation when I set the pericenter to 20 kpc, I get the following snapshot at 300 Myr.

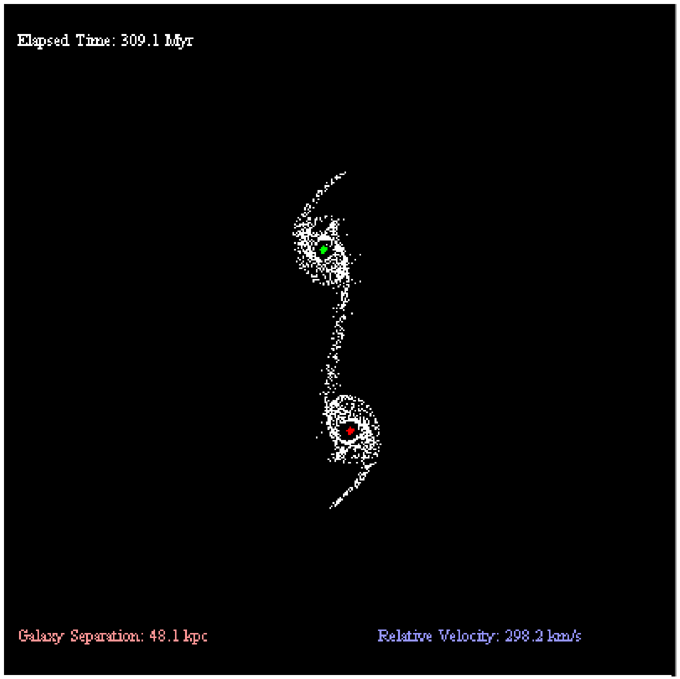
(b) (3 pt) Merger dynamics

For a companion mass that is 1/4 as large, it takes about 3000 Myr to merge.

For a companion mass that is half as large, it takes about 750 Myr to merge.

For the default simulation with friction, it takes about 470 Myr for the galaxies to merge.

For a companion mass that is twice as large, it takes about 470 Myr to merge.



For a companion mass that is 4 times as large, it takes about 625 Myr to merge.

For a companion mass that is 8 times as large, it takes about 1000 years to merge.

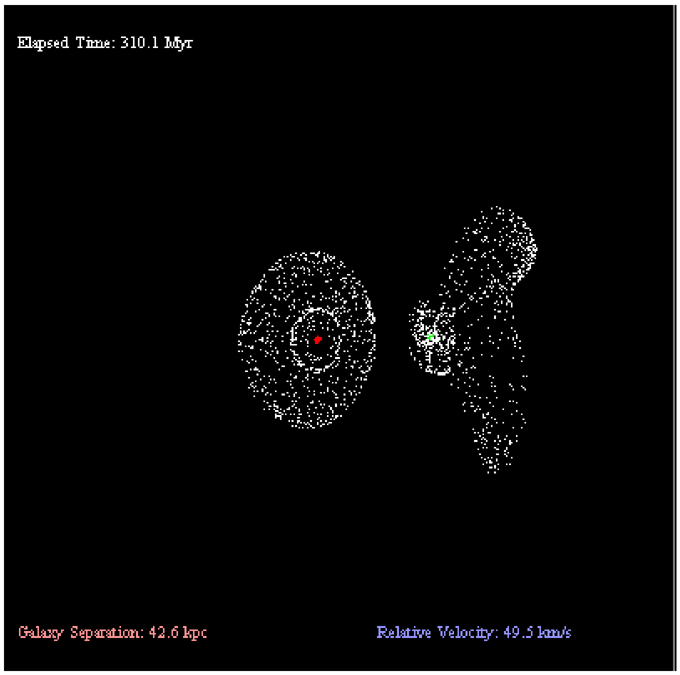
So the fastest mergers seem to occur when the galaxies have about the same mass.

The merging timescale depends on the dynamical friction of the smaller body within the halo of the larger body. When the mass of the smaller one is too small relative to the larger one, the dynamical friction becomes weaker, causing a longer timescale for merging.

When I make the pericenter 15 kpc, the merging timescale is 750 Myr. When I make it 5 kpc, the merging timescale is about 400 Myr. The closer the approach of the two bodies, the larger the density each of them sees of the other body, so the stronger the dynamical friction and shorter timescale for merging.

(c) (3 pt) Real Galaxies:

For the Cartwheel galaxy, I set Peri = 0 kpc, Red Galaxy Mass = 5.0, Number of stars =2000, Friction on, Red Theta = -85 deg, Red phi = -95 deg, Green Theta and Phi = 0. Here is a snapshot at 310 Myr in this simulation. The Green galaxy goes right through the red one, along the angular momentum axis of the red galaxy. The gravity of the green galaxy causes the red one to compress and sends the stars to the center which then go through one another and out in a ring, making the cartwheel shape. Since the green galaxy slices through the red, it gets extremely distorted, more so than in the real galaxy picture.



For the Mice, I ran the standard simulation, but with Green Theta = -45 deg, Peri = 15 kpc, and friction turned on. After 325 Myr, I tilted the simulation so that red is seen nearly edge on, while the tidal tail of green is seen at an angle. The match isn't perfect, but may imply that the other galaxy has a tidal tail seen edge-on.

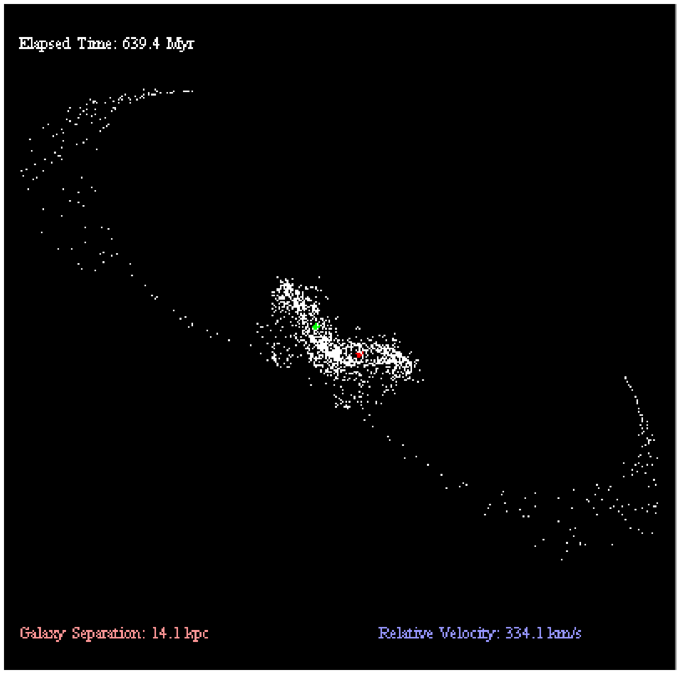
For the Antennae, you have to do a bit of trickery (the initial conditions are actually described in chapter 26, Fig. 26.12). The galaxies look about the same size, and the tidal tails are about equally as strong, but the tails are curving around in the opposite direction. This implies that the two galaxies are tilted with respect to their orbital plane, but

in opposite directions. That way they still have resonant interactions to create strong tails, but the opposite tilt allows the tails to be viewed as curving with opposite handedness. I ran the default simulation with friction, with Red Theta = -45 deg, Green Theta = -45 deg, and Green Phi = 180 deg. At 639 Myr I tilted the simulation until the tails appear to be curving in opposite directions - one is curving towards you and one away, so they are actually pointing in the the same sense in the orbital plane of the galaxies, but from this perspective it looks like they are curving in the opposite direction.

For M 51, I chose Red Theta = -50, Green Theta = -180, Red Galaxy Mass = 2.0, and turned on friction (otherwise same as default). This caused the green galaxy (of lower mass) to undergo weaker tidal distortion due to the retrograde motion of the stars relative to the orbital plane, while the larger galaxy undergoes more tidal distortion leading to a spiral shape due to resonant interaction.

(d) (2 pt) Galaxy Evolution:

Mergers with similar sized galaxies that are nearly head-on, but with disks tilted to one another make good ellipticals. Bad ellipticals form



when the galaxy planes are aligned with the orbital plane, or when one galaxy is much smaller than the other - in these cases the galaxies do not have a triaxial shape like an elliptical (when viewed from various angles).

4. (2 pt) CO 26.2

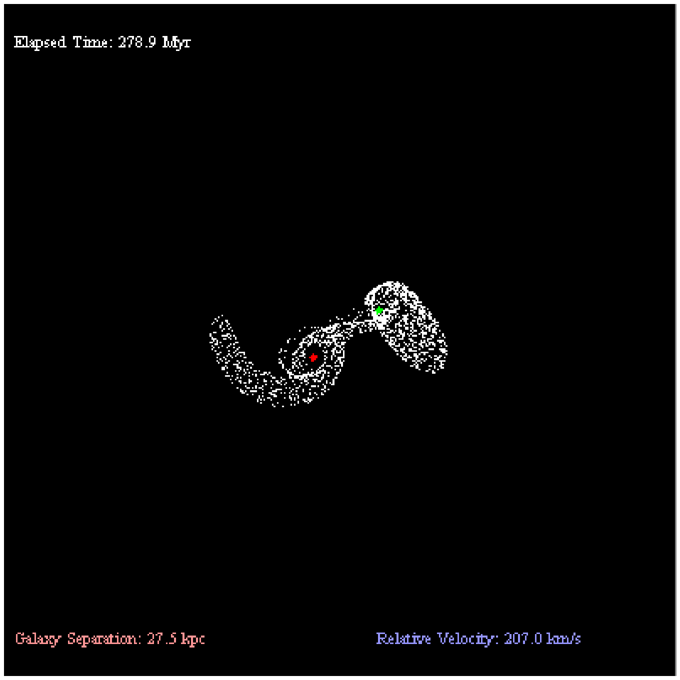
Solution: Using dimensional analysis, we write down the dimensions of each side of the equation:

$$\frac{g^1 cm^1}{s^2} = \left(\frac{cm^3}{s^2}\right)^a cm^b s^{-b} g^c cm^{-3c}. \tag{2}$$

Equating the exponents for g, cm, and s on both sides, we find for grams: $1 = c$, for cm: $1 = 3a + b - 3c = 3a + b - 3$, and for s: $-2 = -2a - b$. Adding the last two equations: $-1 = a - 3$, or $a = 2$. Thus, $b = -2$. So, $f = C\frac{G^2M^2\rho}{v_M^2}$, as in equation (26.1).

5. (2 pt) CO 26.4

Solution: (a) (1 pt) Assuming a flat rotation curve means that the



LMC orbits at 220 km/s. Applying equation 26.2, one finds:

$$t_c = \frac{2\pi 220\text{kms}^{-1}(51kpc)^2}{(23)(6.67d-8cm^3s^{-2}g^{-1})(2\times10^{10}M_\odot)} = 2\times10^9\text{yr}. \tag{3}$$

(b) (1 pt) The text quotes a merger time of 14 Gyr, which is much longer. However, this simple formula gives the right ballpark.

6. (3 pt) CO 26.12 How does this compare to the mass of the Milky Way?

Solution: (a) (2 pt) Combining equations (26.6)-(26.8), and $\mu m_H n = \rho = 3M/(4\pi R^3)$, one finds:

$$t_{cool} = \frac{\mu^2 m_H^2 G 2\pi R^2}{5\Lambda}. \tag{4}$$

The free-fall time is given by the equation in Example 12.2.1: $t_{ff}^2 = 3\pi/(32G\rho)$. Plugging in the above relation for ρ, and setting $t_{ff}^2 = t_{cool}^2$ gives:

$$\frac{4\pi^2 R^3}{32GM} = \frac{\mu^4 m_H^4 G^2 4\pi^2 R^4}{25\Lambda^2}. \tag{5}$$

Solving for mass

$$M = \frac{25\Lambda^2}{32\mu^4 m_H^4 G^3 R}. \tag{6}$$

Larger galaxies have a cooling timescale that is longer than the free-fall timescale, so the gas will start to collapse, but then stop since it cannot cool quickly enough. This is similar to the mass of the Milky Way, but this is something of a coincidence since much of the Milky Way mass is in dark matter, not gas.

(b) (1 pt) Setting $\mu = 1$ and $m_H = 1.67 \times 10^{-24}$ g, I find:

$$M = \frac{25(10^{-37})^2}{32(1.67 \times 10^{-24})^4 (6.67 \times 10^{-11})^3 60 \times 3.08 \times 10^{21}} = 10^{12} M_\odot. \tag{7}$$

I have left the units out of this equation, but have checked that the units agree on both sides of the equation.