ASTR 323 Spring 2009

Problem Set 5 Solutions

23 points total

1. (5 pt) Distance Scales: Assume you have an SDSS-like telescope that you can use to detect and measure stars as faint as V=22.5, and that you are observing towards a region with negligible interstellar extinction.

   a) (2 pt) If the cepheid period-luminosity relation is given by eq. 14.1, how far can you detect a cepheid with a period of 7 days?

   Solution: A Cepheid with a period of 7 days has a V-band magnitude of -3.8 according to eq. 14.1. The distance modulus for a star seen at V=22.5 with this absolute magnitude is: $d = 10^{0.2(22.5-(-3.8)+5)} = 2$ Mpc, assuming no extinction.

   b) (1 pt) How far can you detect an RR Lyrae star (M_V=0.7)?

   Solution: The distance modulus is $d = 10^{0.2(22.5-0.7+5)} = 229$ kpc.

   c) (2 pt) What would be apparent magnitudes of the same cepheid and RR Lyrae stars at the distance corresponding to $z = 0.1$ for $h = 0.71$?

   Solution: For $z = 0.1$ the distance is $d = cz/H_0 = 422$ Mpc **(Note: in the lecture notes I handed out in class there was a typo for the units of H_0 - the correct units are km s^{-1} Mpc^{-1} - please correct this in your notes.)**. At this distance the Cepheid would have a magnitude of $V = 34.3$ while the RR Lyrae would have a magnitude of $V = 38.8$. The former might be detectable by HST (although it would probably be blended together with all the other stars in the galaxy), while the latter would be undetectable. This indicates that other techniques are required for distance measurements at redshift 0.1.

2. (5 pt) Uncertainties in Distance Measurements: Use the relation $D = (L/4\pi F)^{1/2}$ and the error propagation formula: for $f(x, y)$, $\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2$.

a) (1 pt) Suppose I mis-measure the apparent flux from a galaxy by 10%, and that I'm using the Tully-Fisher relationship to derive the galaxy's distance. Assuming that the Tully-Fisher relationship and my measurement of the rotation speed of the galaxy are both perfect, what is the corresponding percentage error I would make in the distance to the galaxy?

Solution: Computing $d = (L/4\pi F)^{1/2}$, $\partial d/\partial F = -\frac{1}{2}d/F$. So, $\sigma_d^2 = \frac{1}{4}(d/F)^2\sigma_F^2$. So, $\sigma_d/d = \frac{1}{2}\sigma_F/F = 5\%$.

b) (2 pt) What is the percentage distance uncertainty which results from the combination of my 10% uncertainty in the measured flux from part (a), and an intrinsic 25% scatter in the luminosity, L, inferred from the velocity via the Tully-Fisher relationship?

Solution: $\sigma_d^2 = \frac{1}{4}(d/F)^2\sigma_F^2 + \frac{1}{4}(d/L)^2\sigma_L^2$. So, $\sigma_d/d = \frac{1}{2}\sqrt{\sigma_F^2/F^2 + \sigma_L^2/L^2} =$ 13%.

c) (2 pt) Assuming that the Tully Fisher relationship is $L \propto V^4$, what is the percentage distance uncertainty which results from the combination of errors from b), AND an additional error of 10% in the rotation speed (e.g. due to inclination effects)?

Solution: Well, the Tully-Fisher relation can be written as $L = bV^4$, where b is a constant of proportionality. The uncertainty on the rotation speed and uncertainty on b (which is the scatter in the Tully-Fisher relation) means: $\sigma_L/L = \sqrt{\sigma_b^2/b^2 + 16\sigma_V^2/V^2} = 0.47$. Folding this through the expression for d in terms of L and F, $\sigma_d/d = 27\%$. This is a huge uncertainty on distance, so the Tully-Fisher relation is not the most reliable distance indicator.

3. (3 pt) CO 27.11

Solution: If the rotation curve is flat, then $M = V^2r/G$ and $\rho = V^2/(4\pi G r^2)$. So, $\partial \ln\rho/\partial \ln r = -2$. If we assume $T = T_0 r^\alpha$, then $\partial \ln T/\partial \ln r = \alpha$. So, from equation (27.17):

$$M = \frac{V^2 r}{G} = -\frac{kT_0 r^\alpha r}{\mu m_H G}(-2 + \alpha) \tag{1}$$

Now, since both sides of this equation are equal, then they should agree at all radii, r, so they should have the same dependence on r. So, $r^1 = r^{\alpha+1}$ or $\boxed{\alpha = 0.}$

4. (8 pt) CO 27.12

Solution: (a) (3 pt) Using eqn. 27.19 and 27.20,

$$1.5{\times}10^{36}\mathrm{W} = \frac{4\pi}{3}(1.5\mathrm{Mpc})^3(3{\times}10^{22}\mathrm{m/Mpc})^3 1.4{\times}10^{-40} n_e^2 (70{\times}10^6\mathrm{K})^{1/2}\mathrm{Wm}^{-3}. \quad (2)$$

A units check shows that this equation has the wrong units; however, the description of equation 27.19 says that the units are written such that n_e has units of m^{-3} and T has units of Kelvin. Solving for electron number density gives $\boxed{n_e = 58\ \mathrm{m}^{-3}.}$ The mass in gas is then given by $M_{gas} = \frac{4\pi}{3} R^3 n_e m_H$ yielding $\boxed{M_{gas} = 2 \times 10^{13} M_\odot.}$

(b) (2 pt) For the old stellar component of a galaxy, a typical mass-to-light ratio is 3 (see table for Milky Way's bulge mass-to-light ratio). This implies a mass in stars of $3.6 \times 10^{12} M_\odot$ which is about 6 times smaller than the mass in gas.

(c) (3 pt) The energy density is: $U = \frac{3}{2}NkT$, while the cooling rate is $\mathcal{L}_{vol} = 1.42 \times 10^{-40} n_e^2 T^{1/2}$ W m^{-3}. Thus, the cooling timescale is $t_{cool} = U/\mathcal{L}_{vol} = 3 \times 10^{17} \mathrm{sec}\, n_e^{-1} T^{1/2} = 1.3 \times 10^{12}$ year. This is about 1000 times as long as the Hubble time (the age of the Universe). This means that the cluster will remain in hydrostatic equilibrium for a long time.

5. (2 pt) CO 27.13

Solution: The crossing time is $2R/\sigma = 2{\times}3\mathrm{Mpc}{\times}3{\times}10^{22}\mathrm{mMpc}^{-1}/977\mathrm{km/s} = 6 \times 10^9$ yr. This is about half of the Hubble time. Thus, if the galaxies were not bound to the cluster they would have departed by now. Hence the argument that dark matter is required to bind the galaxies to the cluster is valid.