1) In words, what is the critical density $\rho_c$?

2) What is the density parameter $\Omega$?

3) What are the four main mass-energy components that contribute to the total density parameter $\Omega_{tot}$?

4) How do we know that baryons alone cannot account for $\Omega_m \sim 0.3$?

5) Why are universes with $\Omega > 1$ called “closed” (assuming $\Omega_\Lambda = 0$)?

6) What is the Hubble law?

7) What are the comoving coordinate $a$ and the scale factor $R$, and how do they relate to the physical separation $r$ between galaxies as a function of time?

8) Briefly explain the origin of the cosmological redshift (i.e. why is the wavelength of a photon that we observe today longer than its wavelength when it was emitted?).

9) Express the redshift we would measure today for a photon which was emitted at time $t$, in terms of the scale factor $R$. Let $t_0$ be the current age of the universe.

10) Why does $\rho_0 = \rho(t)R(t)^3$ for matter?

11) Briefly explain how “standard candles” can be used to measure cosmological parameters.

12) Briefly explain three methods of your choice for determining distances to astronomical objects.

13) In words, why does the value of the Hubble Constant change with time?

14) Briefly explain three main observational results that support the Big Bang theory.

15) How does galaxy clustering depend on galaxy type?

16) What is gravitational lensing? Microlensing?
17) What are gamma-ray bursts?
18) What is the cosmological concordance model?
19) Which cosmological parameter is strongly constrained by the measured abundance of light elements?
20) What problems of the Big Bang theory are addressed by the inflation theory?
21) What are the basic properties of the Cosmic Microwave Background?
22) What happened in the life of the universe when the CMB photons were emitted?
23) Why does the Cosmic Microwave Background support the Big Bang theory?
24) What feature of the Cosmic Microwave Background quantitatively constrains the geometry of the universe?

For the following four questions you will make use of the Friedmann equation for the evolution of the scale factor:

\[
\left( \left( \frac{dR(t)}{dt} \right)^2 - \frac{8\pi G \rho(t)}{3} \right) R^2(t) = \left[ H(t)^2 - \frac{8\pi G \rho(t)}{3} \right] R^2(t) = -kc^2
\]

You may assume for this problem that the universe is flat and \( \Omega_\Lambda = 0 \).

25) Give a physical explanation for the two terms on the left hand side and show that the current value of the critical density is \( \rho_{c,0} = \frac{3H_0^2}{8\pi G} \).

26) Show that for a flat universe, the Friedmann equation can be rewritten as:

\[
\left( \left( \frac{dR(t)}{dt} \right)^2 - \frac{H_0^2}{R(t)} \right) = 0
\]

27) Solve the differential equation from 26) to show that \( R(t) = (3H_0 t/2)^{2/3} \), and explain how it implies that the age of the universe is \( t_0 = \frac{2}{3H_0} \).

28) If you are given \( E(z) = 1/R dR/dt \), how would you compute the age of the universe? (write down and explain the resulting integral expression – you don’t need to evaluate it)