

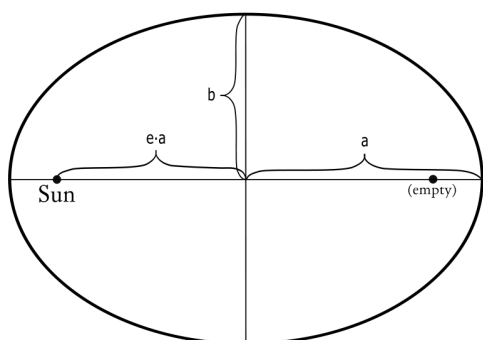
A VERY INCOMPLETE ORBITAL ELEMENTS TUTORIAL

The Keplerian orbital elements for an object are the six parameters that uniquely specify the size, shape and orientation of an orbit and the position of a object along that orbit. The six Keplerian orbital elements are:

<p>a - Semimajor axis e - Eccentricity i - Inclination</p>	<p>Ω - Longitude of the ascending node ω - Argument of the perihelion M - Mean anomaly at epoch</p>
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Kepler's first law says: *The orbit of every planet is an ellipse with the sun at one focus.* The Semimajor axis (**a**) and the eccentricity (**e**) parametrize the size and shape of the ellipse. The units of **a** in our dataset are Astronomical Units (AU) (an AU is the average distance between the Sun and the Earth, 1 AU = 149,597,870,691 m).

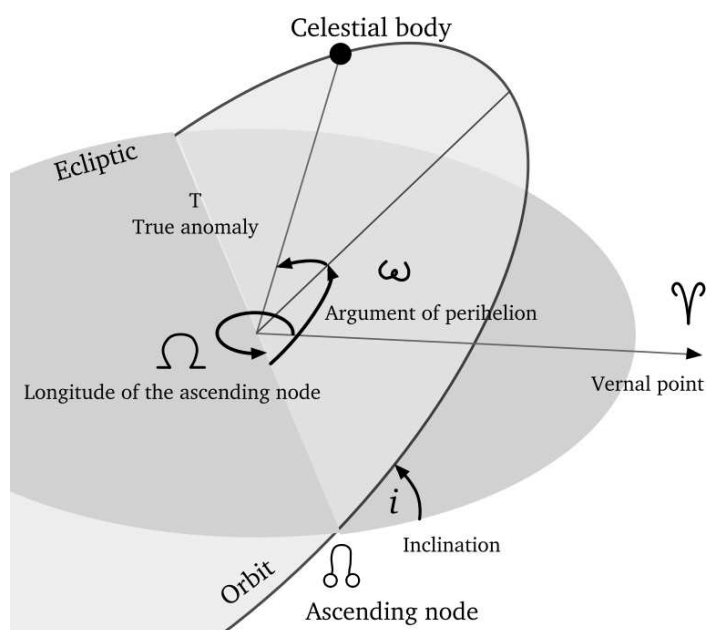
For a closed elliptical orbit (orbits gravitationally bound to the Sun), $e = \sqrt{1 - b^2/a^2}$, where **a** and **b** are the semimajor and semiminor axes (see drawing below). As you can see from the equation when $a = b$, $e = 0$ (a circle), and when $a \gg b$, e approaches 1. When $e = 1$, the orbit is a parabolic orbit (just bound). A hyperbolic orbit is when $e > 1$ (unbounded).



The point in the object's orbit where it is closest to the Sun is called the **perihelion** (periapsis). The farthest point in the orbit is called the **aphelion** (apoapsis). Inspecting the diagram at the right, you can see that the:

$$\begin{aligned} \text{perihelion distance} &= a(1 - e) \\ \text{aphelion distance} &= a(1 + e) \end{aligned}$$

The next three elements fix the orientation of the orbit in space (see the diagram below). These three elements are also known as the *Euler angles*. In order to do this we need to define a frame of reference for our coordinate system. For our solar system the most common frame of reference is to make the X-Y plane correspond to the plane of the ecliptic (the geometric plane containing the mean orbit of the Earth around the Sun). The +X axis points in the direction of the Vernal equinox (the position of the Sun on the first day of spring). It is a "right-handed" coordinate system with the +Z axis pointing in the "northern" direction as seen from the Earth.



The longitude of the ascending node (Ω , $0^\circ < \Omega < 360^\circ$) orients the ascending node with respect to the vernal point. The ascending node is where the orbit crosses the ecliptic plane moving in the +Z direction.

The inclination (**i**, $0^\circ < i < 180^\circ$) orients the orbital plane with respect to the plane of the ecliptic. The plane of the ecliptic is where $i = 0^\circ$.

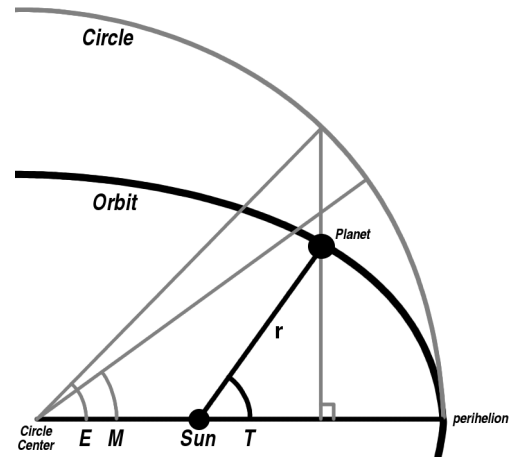
The argument of perihelion (ω , $0^\circ < \omega < 360^\circ$) orients the semimajor axis with respect to the ascending node.

The Mean anomaly at epoch (**M**) is a mess. Think of it as a way to indicate the position of an object along its orbit at a specific time (epoch). In the diagram below, the Mean anomaly is the angle (**M**). Unfortunately, the mean anomaly does not have a nice geometric meaning. We will be using the mean anomaly to calculate two other angles that do have the good geometrical meaning: the *eccentric anomaly* (**E**) and the *true anomaly* (**T**). You can see from the drawing that a few easy relationship hold:

For circular orbits ($e = 0$): $\mathbf{M} \equiv \mathbf{E} \equiv \mathbf{T}$

For all orbits: At Perihelion, $\mathbf{M} = \mathbf{E} = \mathbf{T} = 0^\circ$

For all orbits: At Aphelion, $\mathbf{M} = \mathbf{E} = \mathbf{T} = 180^\circ$



As the names implies the Mean anomaly at epoch specifies the position of an object at a specific time. If we want to know the Mean anomaly at a different time we can find that by the equation:

$$M(t) = M_0 + n(t - t_0) \qquad n = \sqrt{\frac{GM_\odot}{a^3}}$$

where M_0 is the mean anomaly at t_0 , and n is the mean motion. In orbital mechanics, G and M_\odot are typically combined to give the **standard gravitational parameter**, μ . In units of AUs and days:

$$\mu = GM_\odot = 2.9591 \times 10^{-4} \left[\frac{AU^3}{d^2} \right] \Rightarrow n = 1.7202 \times 10^{-2} a^{-3/2} \left[\frac{radians}{day} \right]$$

The Eccentric anomaly (**E**) and the True anomaly (**T**) are related to the Mean anomaly (**M**) as:

$$M = E - e \cdot \sin E \qquad \tan \frac{T}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

The period of the orbit is (Kepler's 3rd law):

$$Period = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}}$$

The orbital position vector (**r**) is the vector connecting the Sun and the object in orbit (see the drawing above) and is related to the eccentric anomaly (**E**):

$$r = a(1 - e \cdot \cos E)$$

The velocity of an object with a given orbital position vector (**r**) is:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)} \left[\frac{AU}{day} \right]$$