Objective

To use the fundamentals of Kepler's laws and knowledge of orbital mechanics to examine the minimum-energy trajectory to Mars; to examine the pros and cons of different orbits.

Introduction*

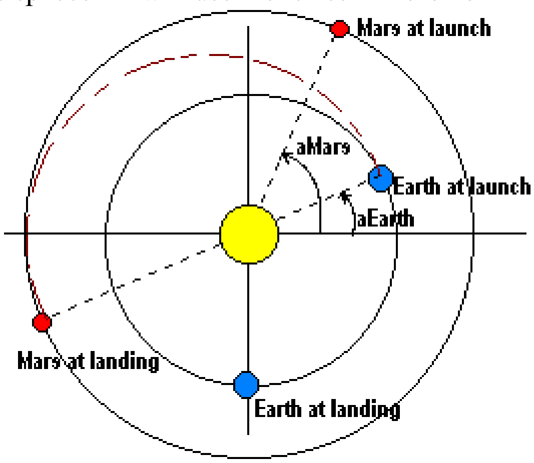
From Earth to Mars

The primary problem to be solved in a manned mission to Mars is the determination of the trajectory to take the spacecraft to Mars. This is fundamental in several ways, for it not only will be a major constraint in choosing the propulsion system, but it will also determine the total time for the journey. The time for the round trip is a very important variable in terms of life support system design, weight of the fully loaded vehicle and medical considerations with respect to the crew. The following sections include the study of the specific problem of an Earth-Mars trip using a minimum energy trajectory.

From Low Earth Orbit to Mars

The most important part of the journey involves transferring from Earth to Mars. The propulsion requirements for this part of the journey are extremely severe and probably determinant in the choice of propulsion system. The most energy efficient way of getting from Earth to Mars is by means of an elliptical, quasi-planetary orbit that is tangential to both the Earth and Mars orbit. This type of orbit is called a Hohmann transfer orbit.

As an initial example, we will use a Hohmann transfer orbit although in a manned mission we would be trying to minimize mission duration. In this particular trajectory, the spacecraft will use the terrestrial orbit's velocity around the Sun to launch it to Mars. An initial rocket burn is required and then in theory the spacecraft can coast to Mars. As it can be seen in the diagram, the total angle relative to the Sun covered by the spacecraft will be 180° .



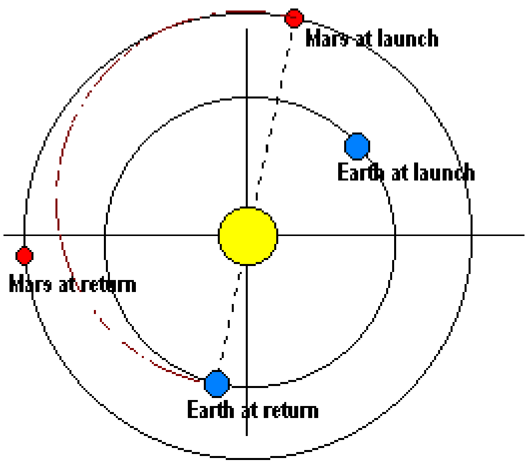
The point where the spacecraft leaves Earth orbit will be the periapsis of the orbit (point in the orbit that is closest to the center, in this case, the Sun.) In this case, as the spacecraft will be traveling in a heliocentric orbit it can directly be called perihelion. When it intercepts the Martian orbit, the spacecraft will be at apoapsis, in this case aphelion, the point in its orbit when it will be farthest away from the Sun. In order to transfer from terrestrial orbit to an orbit that will take the spacecraft farther out from the Sun, as we know, we will need extra speed to attain this higher orbit. Our goal then is to calculate the increase in speed required for the transfer orbit.

The Launch Opportunity

Given the mentioned constraints, the planets must be properly aligned so that when the spacecraft leaves terrestrial orbit with the right impulse given by the rocket burn to enter an orbit such that its perihelion coincides with the radius of the Earth orbit and that aphelion will constitute the radius of the Martian orbit, Mars will be in the correct position so that when the spacecraft arrives in its orbit, Mars is there.

* From Mars Academy -- http://www.marsacademy.com/

Aligning the planets for the comeback.



The figure shows how once again Mars and the Earth have to be in their proper positions to attempt a return trip. When the spacecraft falls back from the Martian orbit, it will enter an elliptical transfer orbit that will encompass half an orbit so that it is tangential to both the terrestrial and Martian orbits. As the Earth moves around the Sun faster than Mars (its angular velocity is nearly double that of Mars), Mars must be advanced with respect to Earth to allow the Earth to intercept the slower spacecraft.

These considerations—Earth/Mars in the proper alignment so that the spaceship intercepts Mars, and Mars/Earth in the proper alignment so that the spaceship intercepts Earth—ensures that the astronauts will be on the Martian surface for about 10 months before the alignments are good enough to get back home. This means that the "least energy trip" will take a **total of 2 1/3 years**.

Orbit information

	Mars	**Venus**	**Earth**
Distance to Sun:	1.52 AU	0.72 AU	1.00 AU
Mean:	227,900,000 km	108,200,000 km	149,600,000 km
Eccentricity:	0.093	0.007	0.017
Period:	686.98 days	224.70 days	365.26 days

Several important conclusions can be drawn from the above table:

Celestial bodies that will influence the trajectory: The orbital mechanics of the mission will be influenced gravitationally by the nearby celestial bodies. In the case of an Earth-Mars mission, the Earth, Mars and the Sun will be primary factors due to mass (in the case of the Sun) and proximity (Earth and Mars) As it can be seen from the table, Venus is sufficiently near to exert its influence. Probably many of the trajectories to be considered will lie very close to Venus and thus be greatly influenced by its gravitational field.

Eccentricity: According to Kepler's Laws, the planets revolve in elliptical orbits, with the Sun is at one of the foci. These elliptical orbits are described by their eccentricity, that is, how much different from a circular orbit they are. In the case of the three planets that influence the trajectory (Earth, Mars, Venus) the values are reduced (maximum value is 0.093 for Mars).

Relative angular speeds of the Earth and Mars : The Martian year is roughly twice as long as the terrestrial year. This means that **Mars' angular speed is half that of the Earth.** The relative angular speeds of the Earth and Mars are very important factors to be taken into account in the determination of the trajectory. When launching from the Earth to Mars, ideally Earth should be much behind in the orbit to allow the spacecraft starting with the Earth's orbital velocity to catch up with the slower Mars. When returning to Earth from Mars our home planet should ideally also be behind to be able to intercept the Mars bound spacecraft in the inner orbit.

Name ____ **Date** ______________

Exercise

1. Calculation of the initial velocity given to each spacecraft leaving Earth due to the Earth's motion: Calculation of Earth's Orbital and Rotational Speeds ("free" orbital energy):

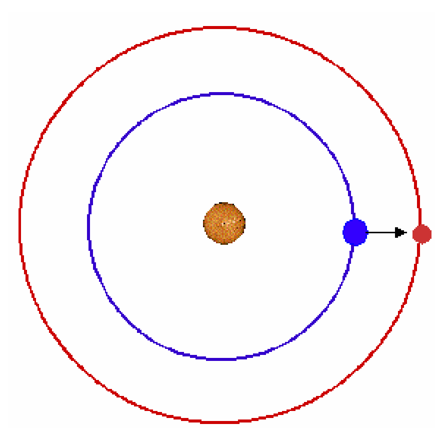
$$\underline{\text{Orbital speed}} = \text{(distance traveled)/(time it takes)} =$$

$$\frac{\text{circumference of Earth's orbit}}{\text{Length of year in seconds}} \frac{2\pi(1.5\times10^8\text{ km})}{(365\text{ dy}\times86400\text{ sec/dy})} = 30\text{ km/sec}$$

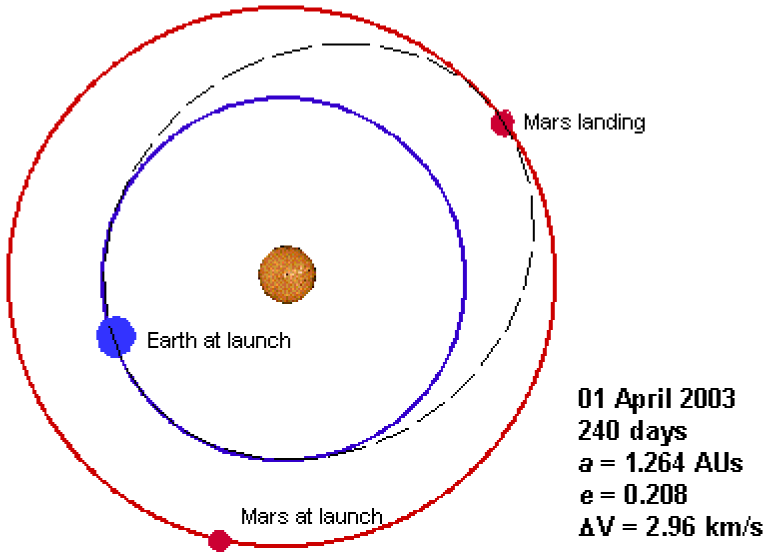
2. Calculate the Earth's rotational speed in km/sec at the equator (Note: radius of Earth at the equator = 6480 km; you need to calculate the circumference and the number of seconds it takes to rotate) – show all calculations:

3. Add up these two velocities for the "free" velocity given to spacecraft if launched in the direction of Earth's orbit, at the equator.

4. Why can't we just use up a whole lot of energy and launch a rocket straight towards Mars when the two planets are the closest to each other, and have a really, really short trip? (There is actually more than 1 reason.)



5. This first figure is an example of a minimum-energy trajectory. The time it will take to get to Mars is 240 days, as shown; ΔV is the **additional** velocity needed to leave Earth's orbit (this is on top of the "free" velocity you calculated above). The location of the Earth at launch is shown. Where will the Earth be when the spaceship lands on Mars?



Mark the location with an "X." Assume that the Earth's orbit is a circle, and that the period is 360days instead of 365 (just makes the calculations easier). **Show all logic here.**

(Hint: if the Earth travels one complete circle in 360 days, what angle will it cover in 240 days?)

6. Let's assume that the spaceship does not land on Mars, and the trip home is the same length of time as the trip to Mars. The astronauts simply wave at the Martians and continue in **the same orbit back** to Earth. a) Approximately how long will this round trip take? b) Where will the Earth be in its orbit after this length of time? Mark this location of the Earth on the above figure with a "Y." c) What kind of problem is posed by this fact when planning a manned mission to Mars.

7. The orbital period you calculated above is not exact due to the approximations in the program used to calculated these values for the trajectories (all orbits are circular). Calculate the more exact orbital period for the spacecraft using Kepler's Third Law: $P^2 = a^3$, where a = 1.264 AUs. Comment on the value you get; that is, does this value make sense when compared to the orbital periods of the Earth and Mars?